SOLVED PAPER

M.Sc. MATHEMATICS (Part II)

NUMERICAL ANALYSIS

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Notes Are Available In Courier Services.

PAPER S/2015

M. Azeem

Numerical Analysis

Q1{a}

Find a root between 0 and 1 of the equation $1+\cos t-4t=$ Correct to three decimals with Secant Method.

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Solution

Let $f(x)=1+\cos t-4t$

$f(0)=1+\cos(0)-4(0)=2>0$

$f(1)=1+\cos(1)-4=-2.4596<$

So the root lies between $(0,1)$

Let $x_0=0=P_0$

$x_1=1=P_1$

Using Secant formula

$$P_{n+1}=P_n-\frac{f(P_n)[P_n-P_{n-1}]}{f(P_n)-f(P_{n-1})} \quad \text{(A)}$$

put $n=1$ in (A)

$$P_2=P_1-\frac{f(P_1)[P_1-P_0]}{f(P_1)-f(P_0)}$$

$$=1-\frac{(-2.4596)(1-0)}{(-2.4596-2)}$$

$=1-0.551529$

$P_2=0.448470$

$f(P_2)=1+\cos(0.4484)-4(0.4484)=0.10727$

Solved by Labeech Ahmad

Azeem

/1

the new interval is [0.448470, 1]

P_2 P_1

$$P_3 = P_2 - \frac{f(P_2)[P_2 - P_1]}{f(P_2) - f(P_1)}$$

$$P_3 = 0.448470 - \frac{(0.10722)[0.448470 - 1]}{[0.10722 - (-2.4596)]}$$

$$= 0.448470 - (-0.02303)$$

$$P_3 = 0.471508$$

$$f(P_3) = 1 + \sin(0.471508) - 4(0.471508)$$

$$= 0.00485 > 0$$

The new interval is (0.471508, 1)

P_3 P_2

put $n=3$ in (A)

$$P_4 = P_3 - \frac{f(P_3)[P_3 - P_2]}{f(P_3) - f(P_2)}$$

$$= 0.471508 - \frac{(0.00485)[0.471508 - 1]}{[0.00485 - (-2.4596)]}$$

$$= 0.471508 - (-0.00104)$$

$$P_4 = 0.47046$$

$$f(P_4) = 1 + \sin(0.47046) - 4(0.47046)$$

$$= 0.009484 > 0$$

The new interval is

(0.47046, 1)

P_4 P_3

$$P_5 = P_4 - \frac{f(P_4)[P_4 - P_3]}{}$$

Solved by

P5

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P6 =

P6

So

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Q5

See

$p_5 = 0.47046 - \frac{(0.009484)[0.47046 - 1]}{[0.009484 - (-2.4596)]}$

$= 0.47046 - (0.020340)$

$p_5 = 0.472494$

$f(p_5) = 1 + 6s(0.472494) - 4(0.472494)$

$= 0.0004600 > 0$

The new interval is

$(0.472494, 1)$

$\downarrow \quad \downarrow$

$p_5 \quad p_4$

$p_6 = p_5 - \frac{f(p_5)[p_5 - p_4]}{f(p_5) - f(p_4)}$

$= 0.472494 - \frac{(0.0004600)[0.472494 - 1]}{[0.0004600 - (-2.4596)]}$

$= 0.472494 - (-0.0000986)$

$p_6 = 0.472592$

So the approximate root correct to three decimal places is $\boxed{0.472}$

Q5(a)

See paper A/2015 Q5(a)

Solved by
Labeeb Ahmad

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PAPER A/2015

Q1(b)

Find a root of $e^x - 3x = 0$ Correct to two decimals with Bisection Method.

Solution

Let $f(x) = e^x - 3x$

$f(0) = e^0 - 3(0) = 1 > 0$

$f(1) = e^1 - 3(1) = -0.2817 < 0$

So the root lie between "0" & "1"

Take a mid point

$$x_0 = \frac{0+1}{2} = 0.5$$

$f(0.5) = e^{0.5} - 3(0.5) = 0.1487 > 0$

There for the root lie between 0.5 and 1

$$x_1 = \frac{0.5+1}{2} = 0.75$$

$f(0.75) = e^{0.75} - 3(0.75) = -0.1329 < 0$

Roots lies b/w 0.5 and 0.75

$$x_2 = \frac{0.5+0.75}{2} = 0.625$$

$f(0.625) = e^{0.625} - 3(0.5625) = -0.1760 < 0$

Solved by:

Laheeb Ahmad

Root lies b/w 0.5 & 0.625

$x_3 = \frac{0.5+0.625}{2} = 0.5625$

$f(0.5625) = e^{0.5625} - 3(0.5625)$

$= 0.0675 > 0$

Root lies between 0.5625 & 0.625

$x_4 = \frac{0.5625+0.625}{2} = 0.59375$

$f(0.59375) = e^{0.59375} - 3(0.59375)$

$= 0.0295 > 0$

Root lies b/w 0.59375 & 0.625

$x_5 = \frac{0.59375+0.625}{2} = 0.6093$

$f(0.6093) = e^{0.6093} - 3(0.6093)$

$= -0.01138 > 0$

Root lie b/w 0.6093 & 0.625

$x_6 = \frac{0.6093+0.625}{2} = 0.61715$

$f(0.61715) = e^{0.61715} - 3(0.61715)$

$= 2.1876 \times 10^{-3} > 0$

Root

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Root

x

f(0

Hence

two

A/2015

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$e^x \sin x$

with

Solu

Solved by

Tahoob Ahmad

Root lies b/w 0.61715 & 0.625

$$x_7 = \frac{0.61715 + 0.625}{2} = 0.6210$$

$f(0.6210) = e^{0.6210} - 3(0.6210)$

$= -2.07 \times 10^{-3} < 0$

Root lie b/w 0.61715 & 0.6210

$$x_8 = \frac{0.61715 + 0.6210}{2} = 0.619075$$

$f(0.619075) = e^{0.619075} - 3(0.619075)$

$= -1.56 \times 10^{-5} < 0$

Hence the root correct to two decimal is **0.61**

1/2015, 5/2015 {05/a}

Q5.(b) Determine a root of $e^x \sin x - \frac{x}{2} = 0$ correct to four decimal with Newton's Raphson method.

Solution

$$e^x \sin x - \frac{x}{2} = 0$$

Let $f(x) = e^x \sin x - \frac{x}{2}$

Solved by
Labeeb Ahmad

$f(\frac{1}{2}) = e^{0.5} \sin(0.5) - \frac{0.5}{2}$

$= 0.5404 > 0$

$f(\pi) = e^{\pi} \sin \pi - \frac{\pi}{2} = -1.5707 < 0$

$f'(x) = e^x \cos x + \sin x \cdot e^x - \frac{1}{2}$

$f'(x) = e^x (\sin x + \cos x) - \frac{1}{2}$

$f'(\pi) = e^{\pi} (\cos \pi + \sin \pi) - \frac{1}{2}$

$= -23.6406$

So Newton Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

put $n=0$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \pi - \frac{f(\pi)}{f'(\pi)}$$

$$= \pi - \frac{(-1.5707)}{(-23.6406)}$$

$x_1 = 3.0751$

Solved by

Abeer Ahmad

$f(x_1) = f(3.0751)$

$= -0.09890$

7.07 < 0

$f'(x) = f'(3.0751)$

$= e^{(3.0751)} [\sin 3.0751 + \cos 3.0751] - \frac{1}{2}$

$-\frac{1}{2}$

$= -20.6655$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$x_2 = 3.0751 - \left[\frac{-0.09890}{-20.6655}\right]$

$\frac{1}{2}$

$x_2 = 3.07031$

$f(x_2) = f(3.07031)$

mula

$= e^{3.07031} \sin(3.07031) - \frac{3.07031}{2}$

$= -0.000415$

$f'(x_2) = e^{3.07031} (\sin 3.07031 + \cos 3.07031) - \frac{1}{2}$

$= -20.4591$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 3.07031 - \frac{(-0.000415)}{(-20.4591)}$

$x_3 = 3.07028$

Solved by Labeeb Ahmad

$f(x_3) = f(3.07028)$

$= e^{3.07028} \cdot \sin(3.07028) - \frac{3.07028}{2}$

$= 0.000198$

$f'(x_3) = f(3.07028)$

$= e^{3.07028}(\sin 3.07028 + \cos 3.07028)^{-\frac{1}{2}}$

$= -20.4578$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 3.07028 - \frac{(0.000198)}{(-20.478)}$

$= 3.0702896$

Hence the approximate root is 3.07028

Solved by
Labeeb Ahmad

Paper A/2014

Q1 (b)

Explain the method of False position. Find a root of $x^3 - 2x - 5$ Correct three decimals with the method of false position.

Solution

Let $f(x) = x^3 - 2x - 5$

$f(2) = 2^3 - 2(2) - 5 = -1 < 0$

$f(3) = 3^3 - 2(3) - 5 = 16 > 0$

So the required Root lies b/w the interval $[2, 3]$

a_0 b_0

$$c_0 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)}$$

$$c_0 = \frac{2f(3) - 3f(2)}{f(3) - f(2)} = \frac{(2)(16) - (3)(-1)}{16 - (-1)}$$

$$= 2.0588 > 0$$

$$f(2.0588) = (2.0588)^3 - 2(2.0588) - 5$$

$$= -0.3910 < 0$$

So the new interval is $[2.0588, 3]$

a_1 b_1

Solved by
Labeeb Ahmad

$$c_1 = \frac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)}$$

$$c_1 = \frac{(2.0588)(16) - (3)(-0.3910)}{16 - (-0.3910)}$$

$$c_1 = \frac{34.1138}{16.3910}$$

$$c_1 = 2.0812$$

$$f(2.0812) = (2.0812)^3 - 2(2.0812) - 5$$

$$= -0.1479 < 0$$

So the new interval is $[2.0812, 3]$

(a_2, b_2)

$$c_2 = \frac{a_2 f(b_2) - b_2 f(a_2)}{f(b_2) - f(a_2)}$$

$$c_2 = \frac{(2.0812)(16) - (3)(-0.1479)}{16 - (-0.1479)}$$

$$c_2 = 2.089615$$

$$f(2.089615) = (2.089615)^3 - 2(2.089615) - 5$$

$$= -0.05494 < 0$$

So the new interval is

$[2.08961, 3]$

Solved by

$$c_3 = \frac{a_3 f(b_3) - b_3 f(a_3)}{f(b_3) - f(a_3)}$$

10)

$$c_3 = \frac{(2.08961)(16) - (3)(-0.05494)}{16 - (-0.05494)}$$

$$c_3 = 2.09272$$

$$f(2.09272) = (2.09272)^3 - 2(2.09272) - 5$$

$$= -0.02036 < 0$$

So the new interval is

$$[2.09272, 3]$$

$$\downarrow a_4 \qquad \downarrow b_4$$

$$c_4 = \frac{a_4 f(b_4) - b_4 f(a_4)}{f(b_4) - f(a_4)}$$

$$= \frac{(2.09272)(16) - (3)(-0.02036)}{16 - (-0.02036)}$$

$$c_4 = 2.09387$$

$$f(2.09387) = (2.09387)^3 - 2(2.09387) - 5$$

$$= -7.56 \times 10^{-3} < 0$$

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Solved by
Labeeb Ahmad

So the new interval is $[2.09387, 3]$

a_5 b_5

$$C_5 = \frac{a_5 f(b_5) - b_5 f(a_5)}{f(b_5) - f(a_5)}$$

$$C_5 = \frac{(2.09387)(16) - (3)(-0.00756)}{16 - (-0.00756)}$$

$$C_5 = 2.09429$$

$$f(2.09429) = (2.09429)^3 - 2(2.09429) - 5$$

$$= -0.00282 < 0$$

So the new interval is

$[2.09429, 3]$

a_6 b_6

$$C_6 = \frac{a_6 f(b_6) - b_6 f(a_6)}{f(b_6) - f(a_6)}$$

$$= \frac{(2.09429)(16) - (3)(-0.00282)}{16 - (-0.00282)}$$

$$C_6 = 2.09444$$

$$f(2.09444) = (2.09444)^3 - 2(2.09444) - 5$$

$$= -0.001137 < 0$$

Solved by:
Labeeb Ahmad

0.9387, 3]

b_5

So the New Interval is

$[2.09444, 3]$

a_7 b_7

$$C_7 = \frac{a_7 f(b_7) - b_7 f(a_7)}{f(b_7) - f(a_7)}$$

$$= \frac{(2.09444)(16) - (3)(-0.001137)}{16 - (-0.001137)}$$

$$C_7 = 2.09450$$

$$f(2.09450) = (2.09450)^3 - 2(2.09450) - 5$$

$$= -0.000526 < 0$$

So the new interval is

$[2.09450, 3]$

a_8 b_8

$$C_8 = \frac{a_8 f(b_8) - b_8 f(a_8)}{f(b_8) - f(a_8)}$$

$$C_8 = \frac{(2.09450)(16) - (3)(-0.000526)}{16 - (-0.000526)}$$

$$C_8 = 2.09452$$

$$f(2.09452) = (2.09452)^3 - 2(2.09452) - 5$$

$$= -2.42 \times 10^{-4} < 0$$

Hence the approximate root is $\boxed{2.0945}$

Solved by
Labeeb Ahmad

Q5(a) M2014 END

Find a root of the equation $x^3 - x - 11 = 0$, in interval (2, 3) correct to three decimals using a numerical technique.

Solution

Note = Here we use any method because here not mentioned any particular method.

Let $f(x) = x^3 - x - 11$

$f(2) = 2^3 - 2 - 11 = -5 < 0$

$f(3) = 3^3 - 3 - 11 = 13 > 0$

Now $x^3 - x - 11 = 0$

$x^3 = 11 + x$

$x = (x + 11)^{1/3} = g(x)$

$g'(x) = \frac{1}{3}(x+11)^{-4/3}$

$g'(2) = \frac{1}{3}(2+11)^{-2/3}$

Solved by
Labeeb Ahmad

$|g'(2)| = 0.06029 < 1$

So the Iteration method is applicable

$\therefore x_{n+1} = (x_n + 11)^{1/3}$

$x_1 = (x_0 + 11)^{1/3}$

$x_1 = (2 + 11)^{1/3}$

$x_1 = 2.3513$

$x_2 = (2.3513 + 11)^{1/3}$

$x_2 = 2.3723$

$x_3 = (2.3723 + 11)^{1/3}$

$x_3 = 2.3735$

$x_4 = (2.3735 + 11)^{1/3}$

$x_4 = 2.3736$

$x_5 = (2.3736 + 11)^{1/3}$

of interval cimals method any

Solved by Labeeb Ahmad

$x_5 = 2.3736$

Here the approximation root upto four decimal places is

2.3736

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PAPER A/2013

Q 2 (a) (any)

Find a root correct four Decimal places of equation $\sin x = 5x - 2$, using numerical method.

Solution *

Let $f(x) = \sin x - 5x + 2$

$f(0) = \sin(0) - 5(0) + 2 = 2 > 0$

$f(1) = \sin(1) - 5(1) + 2 = -2.158 < 0$

Hence the Root of equation is lies between $[0, 1]$.

Now $\sin x = 5x - 2$

$$x = \frac{(\sin x + 2)}{5} = g(x)$$

$$\therefore g(x) = \frac{1}{5}(\sin x + 2)$$

$$g'(x) = \frac{\cos x}{5}$$

$$g'(0) = \frac{\cos(0)}{5} = \frac{1}{5} < 1$$

$$g'(1) = \frac{1}{5}\cos(1) = 0.1080 < 1$$

Hence the iteration method is applicable

Solved by Lubna Ahmad

$$x_{n+1} = \frac{1}{5}(\sin x_n + 2)$$

$$x_1 = \frac{1}{5}(\sin x_0 + 2)$$

$$x_1 = \frac{1}{5}[\sin(0) + 2] = 0.4$$

$$x_2 = \frac{1}{5}[\sin(0.4) + 2] = 0.4778$$

$$x_3 = \frac{1}{5}[\sin(0.4778) + 2] = 0.4919$$

$$x_4 = \frac{1}{5}[\sin(0.4919) + 2] = 0.4944$$

$$x_5 = \frac{1}{5}[\sin(0.4944) + 2] = 0.4949$$

$$x_6 = \frac{1}{5}[\sin(0.4949) + 2] = 0.4949$$

Hence the approximation root upto "4" decimal places is 0.4949

NOTE

In numerical method using trigonometric function calculator must be in Radian Mode.

Solved by

Q4(c) Solve $x^3 - 7x^2 + 14x - 8 = 0$

Solution:

$$x^3 - 7x^2 + 14x - 8 = 0$$

$$x^3 - 8 + 14x - 7x^2 = 0$$

$$(x^3 - 2^3) + (14x - 7x^2) = 0$$

$$(x-2)(x^2+2x+4) + (-7x)(x-2) = 0$$

$$(x-2)[x^2+2x+4-7x] = 0$$

$$(x-2)(x^2-5x+4) = 0$$

$$(x-2)(x^2-4x-x+4) = 0$$

$$(x-2)[x(x-4)-1(x-4)] = 0$$

$$(x-2)(x-4)(x-1) = 0$$

$x-2=0$	$x-4=0$	$x-1=0$
$x=2$	$x=4$	$x=1$

Hence the roots of equation is 1, 2, 4.

$r = \frac{2}{1}$ or $r = \frac{4}{2}$

$r = 2, \quad r = 2$

So solutions are in Geometric progression.

Solved by
Labeeb Ahmad

Paper A/2012 Q3{b}

Using Fix Point method to determine a Solution accurate to with in 10^{-2} for

$3x^2 - e^x$ on $[3, 5]$

Solution

Let $f(x) = 3x^2 - e^x$

$f(3) = 3(3^2) - e^3 = 6.9144 > 0$

$f(5) = 3(5^2) - e^5 = -73.61 < 0$

Now $3x^2 - e^x = 0$

or $e^x = 3x^2$

Taking ln both side

$\ln(e^x) = \ln(3x^2)$

$x \cdot \ln e = \ln 3 + \ln x^2$

$x \cdot 1 = 1.09861 + 2\ln x$

$x = 1.09861 + 2\ln x = g(x)$

$g'(x) = \frac{2}{x}$

$g'(3) = \frac{2}{3} = 0.6666 < 1$

$g'(5) = \frac{2}{5} = 0.4 < 1$

Hence the Fix Point Iteration method is applicable.

Solved by
Labeeb Ahmad

$x_{10} = 1.09861 + 2\ln(3.736484271)$

$= 3.734902558$

$x_{11} = 1.09861 + 2\ln(3.734902558)$

$= 3.734055747$

$x_{12} = 1.09861 + 2\ln(3.734055747)$

$= 3.733602238$

$x_{13} = 1.09861 + 2\ln(3.733602238)$

$= 3.733359319$

$x_{14} = 1.09861 + 2\ln(3.733359319)$

$= 3.733229189$

$x_{15} = 1.09861 + 2\ln(3.733229189)$

$= 3.733159475$

$x_{16} = 1.09861 + 2\ln(3.733159475)$

$= 3.733122128$

Hence the Approximation root is

(3.7331)

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Solved by Labeeb Ahmad

$$x_{n+1} = 1.09861 + 2 \ln x_n$$

Put $n = 0$

$$x_1 = 1.09861 + 2 \ln x_0$$

$$x_0 = 5$$

$$x_1 = 1.09861 + 2 \ln(5)$$

$$= 4.31748$$

$$x_2 = 1.09861 + 2 \ln(x_1)$$

$$= 1.09861 + 2 \ln(4.09861)$$

$$x_2 = 4.02395\,87$$

$$x_3 = 1.09861 + 2 \ln(4.02395\,87)$$

$$x_3 = 3.883142$$

$$x_4 = 1.09861 + 2 \ln(3.883142)$$

$$= 3.811901703$$

$$x_5 = 1.09861 + 2 \ln(3.811901703)$$

$$= 1.09861 + 2 (1.338128199)$$

$$= 3.774868687$$

$$x_6 = 1.09861 + 2 \ln(3.774868687)$$

$$= 3.755343483$$

$$x_7 = 1.09861 + 2 \ln(3.755343483)$$

$$= 3.744971798$$

$$x_8 = 1.09861 + 2 \ln(3.744971798)$$

$$= 3.739440461$$

$$x_9 = 1.09861 + 2 \ln(3.739440461)$$

$$= 3.736484271.$$

PAPER A/2010

5/2009 {Q1(b)}

Q1(b) Find up to 4 d.p the real root of equation $e^x - x^2 - 5 = 0$ by using Newton's Raphson's method.

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Solution

Let $f(x) = e^x - x^2 - 5$

$f(2) = e^2 - 2^2 - 5 = -1.61 < 0$

$f(3) = e^3 - 3^2 - 5 = 6.08 > 0$

Hence our Root lies between in $[2,3]$ because $f(2) > 0$ & $f(3) <$

Now $f(x) = e^x - x^2 - 5$

$f'(x) = e^x - 2x$

$\therefore f'(2) = e^2 - 2(2) = 3.3890$

By Newton Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Solved by Labeeb Ahmad

Put $n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \frac{(-1.61)}{3.3890} = 2.4750$$

$$f(x_1) = f(2.4750) = e^{2.4750} - (2.4750)^2 - 5$$

$$= 0.7560$$

$$f'(x_1) = e^{2.4750} - 2(2.450)$$

$$= 6.9317$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.4750 - \frac{0.7560}{6.9317} = 2.3659$$

$$f(x_2) = f(2.3659)$$

$$= e^{2.3659} - (2.3659)^2 - 5$$

$$= 0.05613$$

$$f'(x_2) = e^{2.3659} - 2(2.3659) = 5.9218$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.3659 - \frac{0.05613}{5.9218}$$

$$x_3 = 2.3564$$

Solved by
Labeeb Ahmad

f(x

f'(x3

x4

x4

x1

f(x4

f'(x4

x5

x

Hence

four

50

$$f(x_3) = e^{2.3564} - (2.3564)^2 - 5 = 0.00027$$

$$f'(x_3) = e^{2.3564} - 2(2.3564) = 5.8400$$

4750)-5

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 2.3564 - \frac{0.00027}{5.8400} = 2.3563$$

$$x_4 = 2.3563$$

$$f(x_4) = e^{2.3563} - (2.3563)^2 - 5 = -3.12 \times 10^{-4}$$

.3659

$$f'(x_4) = e^{2.3563} - 2(2.3563) = 5.8392$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

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$$= 2.3563 - \frac{(-0.00032)}{5.8392}$$

$$x_5 = 2.35635$$

Hence the required root Correct four decimal places is

2.3563

Solved by
Labeeb Ahmad

PAPER A/2009

Q1(b) Solve the equation $2x - 3\sin x - 5 = 0$ upto 5 d-p by using Regula-Falsi method.

Solution

Let $f(x) = 2x - 3\sin x - 5$

$f(2) = 2(2) - 3\sin(2) - 5$

$= -3.7278 < 0$

$f(3) = 2(3) - 3\sin(3) - 5$

$= 0.5766 > 0$

Root of equation lie b/w $[2, 3]$ (a_0, b_0)

$$C_0 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)}$$

$$C_0 = \frac{2(0.5766) - 3(-3.7278)}{(0.5766) - (-3.7278)}$$

$C_0 = 2.8660$

$f(2.8660) = -0.08435 < 0$

Solved by Labeeb Ahmad

09

The new interval in which root lie

$[2.8660, 3]$

a_1 b_1

$$c_1 = \frac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)}$$

$$= \frac{(2.8660)(0.5766) - 3(-0.08435)}{(0.5766) - (-0.08435)}$$

$c_1 = 2.884508$

$f(2.884508) = 2(2.884508) - 3\sin(2.884 - 5$

$= 0.0062296$

Hence the new interval in which root lie $[2.8660, 2.884508]$

a_2 b_2

$$c_2 = \frac{a_2 f(b_2) - b_2 f(a_2)}{f(b_2) - f(a_2)}$$

$$c_2 = \frac{(2.8660)(0.0062296) - (2.884508)(-0.0843}{(0.0062296) - (-0.08435)}$$

$c_2 = 2.883235$

Solved by Labeeb Ahmad

$f(2.883235) = 2(2.883235) - 3\sin(2.883235) - 5$

$= -0.00000617 < 0$

Hence the root lie in new interval $[2.883235, 2.884508]$

$a_3 \qquad b_3$

$$c_3 = \frac{a_3 f(b_3) - b_3 f(a_3)}{f(b_3) - f(a_3)}$$

$$c_3 = \frac{(2.883235)(0.0062296) - (2.884508)(-0.00000617)}{(0.0062296) - (-0.00000617)}$$

$c_3 = 2.883236$

$f(2.883236) = 2(2.883236) - 3\sin(2.883236) - 5$

$= -0.00000352 < 0$

Hence the new interval in which root lie

$[2.883236, 2.884508]$

$a_4 \qquad b_4$

$C_4 =$

$C_4 =$

$C_4 =$

Hence

"5" d

Solved By
Labeeb Ahmad

Sin(2·883230)
5
0
8)(−0.0000617)
0617)
in(2-883236)
− 5
0
ich
8]

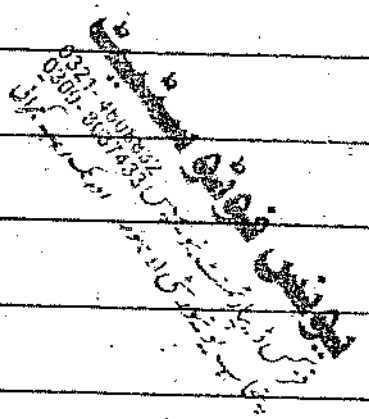
$$C_4 = \frac{a_4 f(b_4) - b_4 f(a_4)}{f(b_4) - f(a_4)}$$

$C_4 = \frac{(2.883236)(0.0062296) - (2.884508)(-0.0000352)}{(0.0062296) - (-0.0000352)}$

$C_4 = 2.8832.36718$

Hence the approximate root of "5" d-p is 2.883236

Solved by
Labeeb Ahmad



PAPER A/2011

Q1 {a} Find an iterative formula to find the cube root of N where N is positive real Number and then calculate cube root of 13.

Solution :- $x = (N)^{1/3}$

$$x^3 = N$$

$$x^3 - N = 0$$

Let $f(x) = x^3 - N$

$$f'(x) = 3x^2$$

Now by using Raphson Formula.

$$x_n = x_{n-1} - f(x_{n-1}) / f'(x_{n-1}) \quad \text{(A)}$$

So $f(x_{n-1}) = x_{n-1}^3 - N$

$$f'(x_{n-1}) = 3x^2 x_{n-1}$$

Put in (A)

$$x_n = x_{n-1} - \frac{(x_{n-1})^3 - N}{3(x_{n-1})^2} \quad \text{(1)}$$

For Cube Root of 13 :-

Put $N = 13$

$n = 1$ & $x_0 = 1$

Solved by

011

$$x_1 = x_0 - \frac{[(x_0)^3 - 13]}{3(x_0)^2}$$

e

$$x_1 = 1 - \frac{[1^3 - 13]}{3(1)^2}$$

oot of

$$x_1 = 1 - (-4) = 5$$

umber

oot

Put $n = 2$ in (1)

$$x_2 = x_1 - \frac{[(x_1)^3 - 13]}{3(x_1)^2}$$

$$= 5 - \frac{(5^3 - 13)}{3(5^2)}$$

$$x_2 = 3.50666$$

Put $n = 3$ in (1)

$$x_3 = x_2 - \frac{[(x_2)^3 - 13]}{3(x_2)^2}$$

(A)

$f'(x_{n-1})$

$$= 3.50666 - \frac{[(3.50666)^3 - 13]}{3(3.50660)^2}$$

$$x_3 = 2.69015$$

(1)

Put $n = 4$ in (1)

$$x_4 = x_3 - \frac{[(x_3)^3 - 13]}{3(x_3)^2}$$

$= 1$

$$= 2.69015 - \frac{[(2.69015)^3 - 13]}{3(2.69015)^2}$$

$$x_4 = 2.39221$$

Solved by
Labeeb Ahmad

put $n=5$ in (1)

$$x_5 = x_4 - \frac{(x_4^3 - 13)}{3x_4^2}$$

$$= 2.39221 - \frac{[(2.39221)^3 - 13]}{3(2.39221)^2}$$

$x_5 = 2.35202$

put $n=6$ in (1)

$$x_6 = x_5 - \frac{(x_5^3 - 13)}{3x_5^2}$$

$$= (2.35202) - \frac{[(2.35202)^3 - 13]}{3(2.35202)^2}$$

$x_6 = 2.35135$

put $n=7$ in (1)

$$x_7 = x_6 - \frac{(x_6^3 - 13)}{3x_6^2}$$

$$= 2.35135 - \frac{[(2.35135)^3 - 13]}{3(2.35135)^2}$$

$x_7 = 2.35133$

Hence the cube root of 13 is 2.3513.

Solved by
Labeeb Ahmed

A/2013, A/2013, S/2011

Convergence of Secent method

The secent formula is

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_n - 1)} \quad (1)$$

Let $x_{n-1} = a + \varepsilon_{n-1}$

$x_n = a + \varepsilon_n$

$x_{n+1} = a + \varepsilon_{n+1}$

Then (1) takes the form

$$a + \varepsilon_{n+1} = a + \varepsilon_n - \frac{(\varepsilon_n - \varepsilon_{n-1}) f(a+\varepsilon_n)}{f(a+\varepsilon_n) - f(a+\varepsilon_{n-1})}$$

Using the Taylor's expensions for $f(a+\varepsilon_n)$ and $f(a+\varepsilon_{n-1})$ we get

$$\varepsilon_{n+1} = \varepsilon_n - \frac{[\varepsilon_n - \varepsilon_{n-1}][f(a) + \varepsilon_n f'(a) + \frac{\varepsilon_n^2}{2} f''(a) + \cdots]}{[f(a) + \varepsilon_n f'(a) + \frac{\varepsilon_n^2}{2} f''(a) + \cdots] - [f(a) + \varepsilon_{n-1} f'(a) + \frac{1}{2}\varepsilon_{n-1}^2 f''(a) + \cdots]}$$

Since $f(a) = 0$ and ε is very very small i-e neglected third and higher order powe

13

Solved by
Labeeb Ahmad

$$\varepsilon_{n+1} = \varepsilon_n - \frac{[\varepsilon_n - \varepsilon_{n-1}]\left[\varepsilon_n f'(a) + \frac{\varepsilon_n^2}{2} f''(a)\right]}{[\varepsilon_n - \varepsilon_{n-1}] f'(a) + \frac{1}{2}[\varepsilon_n^2 - \varepsilon_{n-1}^2] f''(a)}$$

$$= \varepsilon_n - \frac{[\varepsilon_n - \varepsilon_{n-1}]\left[\varepsilon_n f'(a) + \frac{1}{2}\varepsilon_n^2 f''(a)\right]}{[\varepsilon_n - \varepsilon_{n-1}]\left[f'(a) + \frac{1}{2}(\varepsilon_n + \varepsilon_{n-1}) f''(a)\right]}$$

Solved by
Labeeb Ahmad

$\because \varepsilon_n^2 - \varepsilon_{n-1}^2 = (\varepsilon_n - \varepsilon_{n-1})(\varepsilon_n + \varepsilon_{n-1})$

$$= \varepsilon_n - \frac{\left[\varepsilon_n f'(a) + \frac{\varepsilon_n^2}{2!} f''(a)\right]}{\left[f'(a) + (\varepsilon_n + \varepsilon_{n-1})\frac{f''(a)}{2!}\right]}$$

$$= \varepsilon_n - \frac{f'(a)\left[\varepsilon_n + \varepsilon_n^2 f''(a)/2! f'(a)\right]}{f'(a)\left[1 + (\varepsilon_n + \varepsilon_{n-1}) f''(a)/2! f'(a)\right]}$$

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n + \frac{\varepsilon_n^2 f''(a)}{2! f'(a)}\right]\left[1 + \frac{(\varepsilon_n + \varepsilon_{n-1}) f''(a)}{2! f'(a)}\right]^{-1}$$

using Binomial expension

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n + \frac{\varepsilon_n^2 f''(a)}{2! f'(a)}\right]\left[1 - \frac{(\varepsilon_n + \varepsilon_{n-1}) f''(a)}{2! f'(a)} \cdots\right]$$

Neglecting second and higher power

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n + \frac{\varepsilon_n^2 f''(a)}{2! f'(a)}\right]\left[1 - \frac{\varepsilon_n f''(a)}{2! f'(a)} - \frac{\varepsilon_{n-1} f''(a)}{2! f'(a)}\right]$$

$\varepsilon_{n+1} =$

$+$

$\varepsilon_{n+1} =$

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$\varepsilon_{n+1} =$

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$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n - \frac{\varepsilon_n^2 f''(a)}{2! f'(a)} - \frac{\varepsilon_n \varepsilon_{n-1} f''(a)}{2! f'(a)} + \frac{\varepsilon_n^2 f''(a)}{2! f'(a)} - \varepsilon_n^3 \left(\frac{f''(a)}{f'(a)}\right)^2 - \frac{\varepsilon_n \varepsilon_{n-1}}{2!2!}\left(\frac{f''(a)}{f'(a)}\right)^2\right]$$

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n - \frac{\varepsilon_n \varepsilon_{n-1} f''(a)}{2! f'(a)} - \frac{\varepsilon_n^3}{4}\left(\frac{f''(a)}{f'(a)}\right)^2 - \frac{\varepsilon_n \varepsilon_{n-1}}{4}\left(\frac{f''(a)}{f'(a)}\right)^2\right]$$

Neglecting second order power derivative

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n - \varepsilon_n \varepsilon_{n-1} \frac{f''(a)}{2! f'(a)}\right]$$

$$\varepsilon_{n+1} = \varepsilon_n - \varepsilon_n + \varepsilon_n \varepsilon_{n-1} \frac{f''(a)}{2! f'(a)}$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n \varepsilon_{n-1} f''(a)}{2! f'(a)}$$

Solved by
Labeeb Ahmad

$$\varepsilon_{n+1} = K \varepsilon_n \varepsilon_{n-1} \quad \text{where } K = f''(a)/2f'(a)$$

$$\varepsilon_{n+1} \propto \varepsilon_n \varepsilon_{n-1}$$

The relation show that each error is proportional to the product of the previous two errors

PA

Q: L

with L

Solu

L

where

and U

These fo

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 1 & 2 \end{bmatrix}$$

u_{11}

u_{12} =

u_{13} =

u_{11} l_{2}

u_{12} l_{21}

PAPER S/2015

Q1{b} Find the solution of

$$3x + 2y + z = 10$$
$$x + 4y - z = 6$$
$$x + 2y + 5z = 20$$

with LU Decomposition method.

Solution

Here $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & -1 \\ 1 & 2 & 5 \end{bmatrix}$

Let $A = LU$

where L is unit lower triangular matrix and U is upper triangular matrix.

There for.

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & -1 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & a_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & -1 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{11} l_{21} & u_{12} l_{21} + u_{22} & u_{13} l_{21} + u_{23} \\ u_{11} l_{31} & u_{12} l_{31} + u_{22} l_{32} & u_{13} l_{31} + u_{23} l_{32} + u_{33} \end{bmatrix}$$

$u_{11} = 3$ ——— (i)

$u_{12} = 2$ ——— (ii)

$u_{13} = 1$ ——— (iii)

$u_{11} l_{21} = 1$ ——— (iv)

$u_{12} l_{21} + u_{22} = 4$ ——— (v)

Solved by
Labeeb Ahmad

$u_{13} l_{21} + u_{23} = -1$ ——(vi)

$u_{12} l_{31} = 1$ ——(vii)

$u_{11} l_{31} + u_{22} l_{32} = 2$ ——(viii)

$u_{13} l_{31} + u_{23} l_{32} + u_{33} = 5$ ——(ix)

From eqn (iv)

$u_{11} l_{21} = 1$

$l_{21} = 1/u_{11} = 1/3 \Rightarrow$ $\boxed{l_{21} = 1/3}$

From Eqn (v)

$u_{12} l_{21} + u_{22} = 4$

$(2)(1/3) + u_{22} = 4$

$u_{22} = 4 - \frac{2}{3} = 10/3$

$\Rightarrow$ $\boxed{u_{22} = 10/3}$

From Eqn (vi)

$u_{13} l_{21} + u_{23} = -1$

$(1)(1/3) + u_{23} = -1$

$u_{23} = -1 - \frac{1}{3} = -2/3$

$\Rightarrow$ $\boxed{u_{23} = -2/3}$

From Eqn (vii) $u_{11} l_{31} = 1$

$3 \cdot l_{31} = 1$

$\Rightarrow$ $\boxed{l_{31} = 1/3}$

From Eqn (viii)

$u_{11} l_{31} + u_{22} l_{32} = 2$

$(3)\left(\frac{1}{3}\right) + \left(\frac{10}{3}\right) \cdot l_{32} = 2$

$1 + \frac{10}{3} l_{32} = 2$

From u_{13} (i) $\Rightarrow$

So $L =$

And $U =$

Be

$\Rightarrow$

Solved by
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$\frac{10}{3}\ l_{32} = 2 - 1 = 1$

$\Rightarrow \boxed{l_{32} = 3/10}$

From Equation (ix)

$u_{13} l_{31} + u_{23} l_{32} + u_{33} = 5$

$(1)(\frac{1}{3}) + (-\frac{2}{3})(\frac{3}{10}) + u_{33} = 5$

$\frac{1}{3} + (-\frac{1}{5}) + u_{33} = 5$

$u_{33} = 5 - \frac{1}{3} + \frac{1}{5}$

$\Rightarrow \boxed{u_{33} = 73/15}$

So

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 1/3 & 3/10 & 1 \end{bmatrix}$$

And

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 10/3 & -3/3 \\ 0 & 0 & 73/15 \end{bmatrix}$$

Because $AX = B$

$LUx = B$

Let $Ux = Y$

$LY = B$ where $Y = [y_1\ y_2\ y_3]$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ +1/3 & 1 & 0 \\ 1/3 & 3/10 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 20 \end{bmatrix}$$

Solved by
Labeeb Ahmad

$$\begin{bmatrix} y_1 \\ y_1/3 + y_2 \\ y_1/3 + 3y_2/10 + y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 20 \end{bmatrix}$$

$\Rightarrow \boxed{y_1 = 10}$

$\frac{y_1}{3} + y_2 = 6$

$y_2 = 6 - \frac{y_1}{3} = 6 - \frac{10}{3} = \frac{8}{3}$

$\Rightarrow \boxed{y_2 = 8/3}$

$\frac{y_1}{3} + \frac{3y_2}{10} + y_3 = 20$

$y_3 = 20 - \frac{y_1}{3} - \frac{3y_2}{10}$

$y_3 = 20 - \frac{10}{3} - \frac{3}{10}\left(\frac{8}{3}\right)$

$\Rightarrow \boxed{y_3 = 238/15}$

Now $Ux = Y$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 10/3 & -2/3 \\ 0 & 0 & 73/15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8/3 \\ 238/15 \end{bmatrix}$$

$\frac{73}{15} z = \frac{238}{15}$

$\boxed{z = 238/73}$

$\frac{10y}{3} - \frac{2z}{3} = \frac{8}{3}$

$10y = 8 + 2z$

Solved by
Labeeb Ahmad

$$10y = 8 + 2\left(\frac{238}{73}\right)$$

$$10y = \frac{1060}{73}$$

$$\boxed{y = 106/73}$$

$$3x + 2y + z = 10$$

$$3x = 10 - z - 2y$$

$$3x = 10 - \frac{238}{73} - 2\left(\frac{106}{73}\right)$$

$$3x = 10 - \frac{238}{73} - \frac{212}{73}$$

$$3x = 280/73$$

$$\boxed{x = \frac{280}{219}}$$

Hence $x = \frac{280}{219}$, $y = \frac{106}{73}$, $z = \frac{238}{73}$

PAPER A 5/2015 Q5(b)

Calculate five iteration of Jacobi met to find the solution of

$$4x - 2y + z = 3$$

$$2x + 3y - z = 6$$

$$x + 3y + 7z = 11$$

Solved by Labeeb Ahma

Solution

$$x = \frac{3+2y-z}{4} \quad \text{---(i)}$$

$$y = \frac{6-2x+z}{3} \quad \text{---(ii)}$$

$$z = \frac{11-x-3y}{7}$$

1st Iteration

Take initial approximation

$$y_0 = z_0 = 0$$

$$x_1 = \frac{3+2(0)-(0)}{4} = \frac{3}{4} = 0.75$$

$$x_0 = x_1 \quad , \quad z = 0$$

$$y_1 = \frac{6-2(0.75)+0}{3} = 1.5$$

$$x_0 = x_1 \text{ and } y_0 = y_1$$

$$z_1 = \frac{11-0.75-3(1.5)}{7} = 0.8214$$

2nd Iteration

$$y_1 = y_1 \quad \& \quad z_1 = z_1$$

$$x_2 = \frac{3+2(1.5)-(0.8214)}{4} = 1.29465$$

$$y_2 = \frac{6-2(x_2)+z_1}{3} = \frac{6-2(1.29465)+0.8214}{3}$$

$$y_2 = 1.4107$$

Solved by
Labeeb Ahmad

$$z_2 = \frac{11 - x_2 - 3y_2}{7}$$

$$z_2 = \frac{11 - 1.29465 - 3(1.4107)}{7}$$

$$z_2 = 0.7818$$

3rd Iteration

$$x_3 = \frac{3 + 2y_2 - z_2}{4} = \frac{3 + 2(1.4107) - 0.78}{4}$$

$$x_3 = 1.2598$$

$$y_3 = \frac{6 - 2x_3 + z_2}{3}$$

$$= \frac{6 - 2(1.2598) + (0.7818)}{3}$$

$$y_3 = 1.4206$$

$$z_3 = \frac{11 - x_3 - 3y_3}{7}$$

$$= \frac{11 - (1.2598) - 3(1.4206)}{7}$$

$$z_3 = 0.7825$$

4th Iteration

Solved by
Labeeb Ahmad

$$x_4 = \frac{3 + 2y_3 - z_3}{4}$$

$$x_4 = \frac{3 + 2(1.4206) - (0.7825)}{4}$$

$x_4 = 1.2646$

$y_4 = \frac{6 - 2x_4 + z_3}{3} = \frac{6 - 2(1.2646) + (0.78}{3}$

$y_4 = 1.4177$

$z_4 = \frac{11 - x_4 - 3y_4}{7} = \frac{11 - (1.2646) - 3(1.477}{7}$

$z_4 = 0.7831$

5th Iteration

$x_5 = \frac{3 + 2y_4 - z_4}{4} = \frac{3 + 2(1.4177) - (0.783}{4}$

$x_5 = 1.2630$

$y_5 = \frac{6 - 2x_5 + z_4}{3} = \frac{6 - 2(1.2630) + (0.78}{3}$

$\therefore y_5 = 1.41903$

$z_5 = \frac{11 - x_5 - 3y_5}{7}$

$z_5 = \frac{11 - (1.2630) - 3(1.41903)}{7}$

$z_5 = 0.7828$

Written by
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Mob:0341-7224005

PAPER A/2015

Q4{a} Calculate four Iteration of Gauss sedial method to approximate the solution of

$$3x + 4y + 15z = 54.80$$

$$x + 12y + 3z = 39.66$$

$$10x + y - 2z = 7.74$$

Solution

Re-arrange the Equations.

$$10x + y - 2z = 7.74$$

$$x + 12y + 3z = 39.66$$

$$3x + 4y + 15z = 54.80$$

$$|10| > |1| + |-2|$$

$$|12| > |1| + |3|$$

$$|15| > |3| + |4|$$

So Gauss-Seidal method is applicable

$$x = \frac{7.74 - y + 2z}{10} \quad \text{(i)}$$

$$y = \frac{39.66 - x - 3z}{12} \quad \text{(ii)}$$

$$z = \frac{54.80 - 3x - 4y}{15} \quad \text{(iii)}$$

Take initial approximation as

$$y_0 = z_0 = 0$$

1st Iteration

Solved by
Labeeb Ahmad

$$x_1 = \frac{7.74 - 0 + 2(0)}{10} = 0.774$$

$$y_1 = \frac{39.66 - x_1 - 3(z_0)}{12}$$

$$y_1 = \frac{39.66 - 0.774 - 3(0)}{12} = 3.2405$$

$$z_1 = \frac{54.80 - 3(0.774) - 4(3.2405)}{15}$$

$$z_1 = 2.6344$$

2nd Iteration

$$x_2 = \frac{7.74 - y_1 + 2z_1}{10}$$

$$x_2 = \frac{7.74 - 3.2405 + 2(2.6344)}{10}$$

$$x_2 = 0.97683$$

$$y_2 = \frac{39.66 - x_2 - 3z_1}{12}$$

$$y_2 = \frac{39.66 - (0.97683) - 3(2.6344)}{12}$$

$$y_2 = 2.5649$$

$$z_2 = \frac{54.80 - 3x_2 - 4y_2}{15}$$

$$z_2 = \frac{54.80 - 3(0.97683) - 4(2.5649)}{15}$$

$$z_2 = 2.7739$$

3rd Iteration

$$x_3 = \frac{7.74 - y_2 + 2z_2}{10}$$

4th

Solved by
Labeeb Ahmad

$$x_3 = \frac{7.74 - (2.5649) + 2(2.7739)}{10}$$

$$x_3 = 1.07229$$

$$y_3 = \frac{39.66 - x_3 - 3z_1}{12}$$

$$= \frac{39.66 - 1.07229 - 3(2.7739)}{12}$$

$$y_3 = 2.5221$$

$$z_3 = \frac{54.80 - 3x_3 - 4y_3}{15}$$

$$z_3 = \frac{54.80 - 3(1.07229) - 4(2.5221)}{15}$$

$$z_3 = 2.7662$$

4th Iteration

$$x_4 = \frac{7.74 - y_3 + 2z_3}{10}$$

$$x_4 = \frac{7.74 - 2.5221 + 2(2.7662)}{10}$$

$$x_4 = 1.07503$$

$$y_4 = \frac{39.66 - x_4 - 3z_3}{12}$$

$$y_4 = \frac{39.66 - (1.07503) - 3(2.7662)}{12}$$

$$y_4 = 2.5238$$

$$z_4 = \frac{54.80 - 3x_4 - 4y_4}{15}$$

$$z_4 = \frac{54.80 - 3(1.07503) - 4(2.5238)}{15}$$

$$z_4 = 2.7652$$

Solved by Labeeb Ahmad

PAPER A/2014

Q3{a} Calculate four iteration of Gauss seidel Method to approximate the Solution of

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

Solution :-

This Question is Solved in Notes (Sir Ajmal Sahib) And Also Example 4.20 in Book Numerical Methods (V-N-Ve Anurthy)

PAPER A/2013

Q3{a} Solve the system of Linear Equations

$$2x + 3y + 8z = 1$$
$$11x + 2y + z = 15$$
$$x + 10y + 2z = 16$$

Correct to three decimal Places using Gauss Sedial method.

Solved by
Labeeb Ahmad

Solution

Re arrange the Equations

$$11x + 2y + z = 15$$

$$x + 10y + 2z = 16$$

$$2x + 3y + 8z = 1$$

So $|11| > |2| + |1|$

$|10| > |1| + |2|$

$|8| > |2| + |3|$

So Gauss Seidal Method is Applicable

$$x = \frac{15 - 2y - z}{11} \quad \text{(i)}$$

$$y = \frac{16 - x - 2z}{10} \quad \text{(ii)}$$

$$z = \frac{1 - 2x - 3y}{8} \quad \text{(iii)}$$

Take initial approximation is

$$y_0 = z_0 = 0$$

Solved by
Labeeb Ahmad

1st Iteration

$$x_1 = \frac{15 - 2(0) - 0}{11} = 1.3636$$

$$y_1 = \frac{16 - x_1 - 2z_0}{10}$$

$$y_1 = \frac{16 - 1.3636 - 2(0)}{10} = 1.4636$$

$$z_1 = \frac{1 - 2x_1 - 3y_1}{8}$$

$$z_1 = \frac{1 - 2(1.3636) - 3(1.4636)}{8}$$

$$z_1 = -0.7647$$

2nd Iteration

$$x_2 = \frac{15 - 2y_1 - z_1}{11}$$

$$x_2 = \frac{15 - 2(1.46364) - (-0.7647)}{11}$$

$$x_2 = 1.167038$$

4th

$$y_2 = \frac{16 - x_2 - 2z_1}{10}$$

$$y_2 = \frac{16 - (1.67038) - 2(-0.7647)}{10}$$

$$y_2 = \frac{15.85902}{10}$$

$$y_2 = 1.585902$$

$$z_2 = \frac{1 - 2x_2 - 3y_2}{8}$$

$$z_2 = \frac{1 - 2(1.167038) - 3(1.585902)}{8}$$

$$z_2 = -0.7614$$

Solved by
Labeeb Ahmad

3rd Iteration

$$x_3 = \frac{15 - 2y_2 - z_2}{11}$$

$$x_3 = \frac{15 - 2(1.585902) - (-0.7614)}{11}$$

5t

$$x_3 = 1.1445090$$

$$y_3 = \frac{16 - x_3 - 2z_2}{10}$$

$$y_3 = \frac{16 - (1.445090) - 2(-0.7614)}{}$$

$y_3 = 2.0097$

$$z_3 = \frac{1 - 2x_3 - 3y_3}{8}$$

$$z_3 = \frac{1 - 2(1.445090) - 3(2.0097)}{8}$$

$$z_3 = -0.98991$$

4th - Iteration

$$x_4 = \frac{15 - 2y_3 - z_3}{11}$$

$$x_4 = \frac{15 - 2(2.0097) - (-0.98991)}{11}$$

$$x_4 = 1.08822$$

$$y_4 = \frac{16 - x_4 - 2z_3}{10}$$

$$y_4 = \frac{16 - (1.08822) - 2(-0.98991)}{10}$$

$$y_4 = 1.68916$$

$$z_4 = \frac{1 - 2x_4 - 3y_4}{8}$$

$$z_4 = \frac{1 - 2(1.08822) - 3(1.68916)}{8}$$

$$z_4 = -0.78049$$

5th Iteration

$$x_5 = \frac{15 - 2y_4 - z_4}{11}$$

$$x_5 = \frac{15 - 2(1.68916) - (-0.78049}{11}$$

$x_5 = 1.12747$

Solved by
Labeeb Ahmad

$$y_5 = \frac{16 - x_5 - 2z_4}{10}$$

$$y_5 = \frac{16 - 1.12747 - 2(-0.77809)}{10}$$

$$y_5 = 1.643433$$

$$z_5 = \frac{1 - 2x_5 - 3y_5}{8}$$

$$= \frac{1 - 2(1.12747) - 3(1.643433)}{8}$$

$$z_5 = -0.773154$$

6th Iteration

$$x_6 = \frac{15 - 2y_5 - z_5}{11}$$

$$= \frac{15 - 2(1.643433) - (-0.7731)}{11}$$

$$= \frac{12.4862}{11}$$

$$x_6 = 1.13511$$

$$y_6 = \frac{16 - x_6 - 2z_5}{10}$$

$$y_6 = \frac{16 - (1.13511) - 2(-0.773154)}{10}$$

$$y_6 = 1.64111$$

$$z_6 = \frac{1 - 2x_6 - 3y_6}{8}$$

$$z_6 = \frac{1 - 2(1.13511) - 3(1.64111)}{8}$$

$$z_6 = -0.77419$$

7th

x

x

2

x

y

y

z

8th

Solved by Labook

7th Iteration

$$x_7 = \frac{15 - 2y_6 - z_6}{11}$$

$$x_7 = \frac{15 - 2(1.64111) - (-0.77419)}{11}$$

$$x_7 = \frac{12.49197}{11}$$

$$x_7 = 1.13563$$

$$y_7 = \frac{16 - x_7 - 2z_6}{10}$$

$$y_7 = \frac{16 - 1.13563 - 2(-0.77419)}{10}$$

$$y_7 = 1.64127$$

$$z_7 = \frac{1 - 2x_7 - 3y_7}{8}$$

$$z_7 = \frac{1 - 2(1.1356) - 3(1.64127)}{8}$$

$$z_7 = -0.77437$$

8th Iteration

$$x_8 = \frac{15 - 2y_7 - z_7}{11}$$

$$x_8 = \frac{15 - 2(1.64127) - (-0.77437)}{11}$$

$$x_8 = 1.13562$$

$$y_8 = \frac{16 - x_8 - 2z_7}{10}$$

$$y_8 = \frac{16 - (1.1356) - 2(-0.77437)}{10}$$

$$y_8 = 1.64131$$

Solved by
Naheed Ahmad

$$z_8 = \frac{1 - 2x_8 - 3y_8}{8}$$

$$z_8 = \frac{1 - 2(1.13562) - 3(1.64131)}{8}$$

$$z_8 = -0.7743$$

Hence the Approximate value is

$x = 1.1356$ $y = 1.6413$ $z = -0.7743$

Written by
Labeeh Ahmad
Mob:0341-7224005

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PAPER A/2013 {Q4, a} , A/2014 {Q4, b}

3) Calculate seven Iterations of Power method with scalling to approximate eigen vector of the matrix A. Also find Dominant eigen value of the matrix A.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

Solution By taking initial vector

$$y^{(0)} = [1, 1, 1]^T$$

1st iteration

$$y^{(1)} = Ay^{(0)}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 \\ -2+1+2 \\ 1+3+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$$

$$y^{(1)} = 5 \begin{bmatrix} 0.6 \\ 0.2 \\ 1 \end{bmatrix} = K_1 B_1$$

Solved by Labeeb Ahmad

2nd Iteration

4th

$$y^{(2)} = AB_1$$

$$y^{(2)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2.2 \end{bmatrix}$$

$$= 2.2 \begin{bmatrix} 0.4545 \\ 0.4545 \\ 1 \end{bmatrix} = K_2 B_2$$

5th

3rd Iteration

$$y^{(3)} = AB_2$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.4545 \\ 0.4545 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.3635 \\ 1.5455 \\ 2.818 \end{bmatrix} = 2.818 \begin{bmatrix} 0.4838 \\ 0.5484 \\ 1 \end{bmatrix}$$

$$y^{(3)} = K_3 B_3$$

Solved by
Labeeb Ahmad

4th Itration

$$y^{(4)} = AB_3$$

$$y^{(4)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.4883 \\ 0.5484 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5748 \\ 1.5878 \\ 3.1255 \end{bmatrix}$$

$$y^{(4)} = 3.1255 \begin{bmatrix} 0.504 \\ 0.508 \\ 1 \end{bmatrix} = K_4 B_4$$

5th Itration

$$y^{(5)} = AB_4$$

$$y^{(5)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.504 \\ 0.508 \\ 1 \end{bmatrix}$$

$$y^{(5)} = \begin{bmatrix} 1.52 \\ 1.5 \\ 3.028 \end{bmatrix}$$

$$= 3.028 \begin{bmatrix} 0.501 \\ 0.495 \\ 1 \end{bmatrix}$$

$$= K_5 B_5$$

Solved by
Labeeb Ahmad

6th Iteration

$$y^{(6)} = AB5$$

$$y^{(6)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.501 \\ 0.495 \\ 1 \end{bmatrix}$$

$$y^{(6)} = \begin{bmatrix} 1.491 \\ 1.493 \\ 2.986 \end{bmatrix}$$

$$= 2.986 \begin{bmatrix} 0.499 \\ 0.5 \\ 1 \end{bmatrix} = K_6 B_6$$

Here and

Conv

7th Iteration

$$y^{(7)} = AB6$$

$$y^{(7)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.499 \\ 0.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.499 \\ 1.502 \\ 2.999 \end{bmatrix}$$

$$= 2.999 \begin{bmatrix} 0.499 \\ 0.500 \\ 1 \end{bmatrix} = K_7 B_7$$

8th Iteration

$$y^{(8)} = AB7$$

Solved by
Labeeb Ahmad

$$A^{(6)} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0.499 \\ 0.500 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.499 \\ 1.502 \\ 2.999 \end{bmatrix}$$

$$= 2.999 \begin{bmatrix} 0.4998 \\ 0.500 \\ 1 \end{bmatrix} = K_8 B_8$$

Here $K_8 = 2.999$ is the Eigen value and $B_8 = [0.4998 \quad 0.500 \quad 1]^T$ be the Corresponding Eigen vector.

Solved by
Labeeb Ahmad

F

Q6

Solution

$f(x$

$f_0 = f$

$f_1 = f$

$f_2 = f$

$f_3 = f$

$$\int_2^5 \frac{1}{x}$$

PAPER 5/2015

Q6 Evaluate $\int_2^5 \frac{1}{x}\,dx$ using four points Simpson 3/8th Rule

Solution

$n = 3$

$$h = \frac{b-a}{n} = \frac{5-2}{3} = 1$$

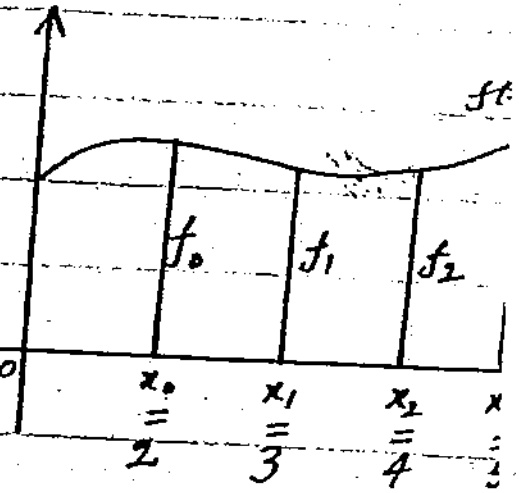
$$f(x) = \frac{1}{x}$$

$$f_0 = f(x_0) = f(2) = \frac{1}{2}$$

$$f_1 = f(3) = \frac{1}{3}$$

$$f_2 = f(4) = 1/4$$

$$f_3 = f(5) = 1/5$$



$$\int_2^5 \frac{1}{x}\,dx = \frac{3}{8}h\left[f_0 + 3f_1 + 3f_2 + f_3\right]$$

$$= \frac{3}{8}(1)\left[\frac{1}{2} + 3\cdot\frac{1}{3} + 3\cdot\frac{1}{4} + \frac{1}{5}\right]$$

$$= \frac{3}{8}\left[2.45\right]$$

$$= 0.91875$$

Solved by
Labeeb Ahmad

PAPER A/2015

Q7: Evaluate $\int_2^5 \sqrt{x+49}\,.dx$ using 5 points Simpson Rule.

Solution:- Here: $n=4$

$h = \frac{b-a}{n} = \frac{5-2}{4} = \frac{3}{4} = 0.75$

$f(x) = \sqrt{x+49}$

$f_0(x_0) = f(2)$

$= \sqrt{2+49} = 7.1414$

$f_1 = f(x_1)$

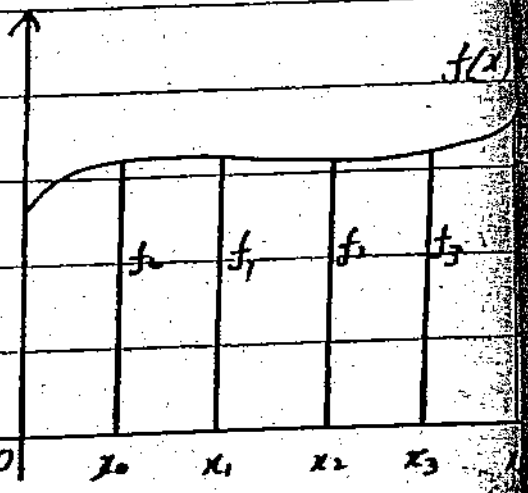
$= \sqrt{\frac{11}{4}+49} = 7.1937$

$f_2 = f(x_2)$

$= \sqrt{\frac{7}{2}+49} = 7.2456$

$f_3 = f(x_3) = \sqrt{\frac{17}{4}+49} = 7.2972$

$f_4 = f(x_4) = \sqrt{5+49} = 7.3484$



Simpson 1/3 Rule

Solved by
Labeeb Ahmad

$$\int_2^5 \sqrt{x+49}\,.dx = \frac{h}{3}\left[(f_0+f_4)+4(f_1+f_3)+2(f_2)\right]$$

Simp

$\int_2^5 \sqrt{x+$

$= \frac{3}{8}(0.7$

$= (0.2$

$= (0$

$= 2.$

$$\int_2^5 \sqrt{x+49}\,dx = \frac{0.75}{3}\Big[(7.1414 + 7.3484) + 4(7.1937 + 7.2972) + 2(7.2456)\Big]$$

$$= (0.25)\Big[(14.4898) + 4(14.4909) + 2(7.2456)\Big]$$

$$= (0.25)(86.9446)$$

$$= 21.73615$$

Solved by Labeeb Ahmad

Simpson 3/8 Rule

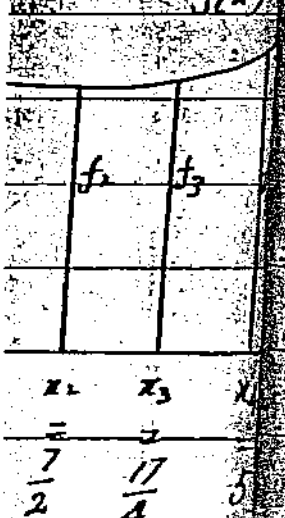
$$\int_2^5 \sqrt{x+49}\,dx = \frac{3h}{8}\Big[(f_0 + f_4) + 3(f_1 + f_2) + 2(f_3)\Big]$$

$$= \frac{3}{8}(0.75)\Big[(7.1414 + 7.3484) + 3(7.1937 + 7.2456) + 2(7.2972)\Big]$$

$$= (0.28125)\Big[(14.4898) + 3(14.4393) + 2(7.2972)\Big]$$

$$= (0.28125)(72.4021)$$

$$= 20.36309063$$



$h = \frac{3}{4} = 0.75$

d by
Ahmad

$4(f_1 + f_3)$

$f_2)\Big]$

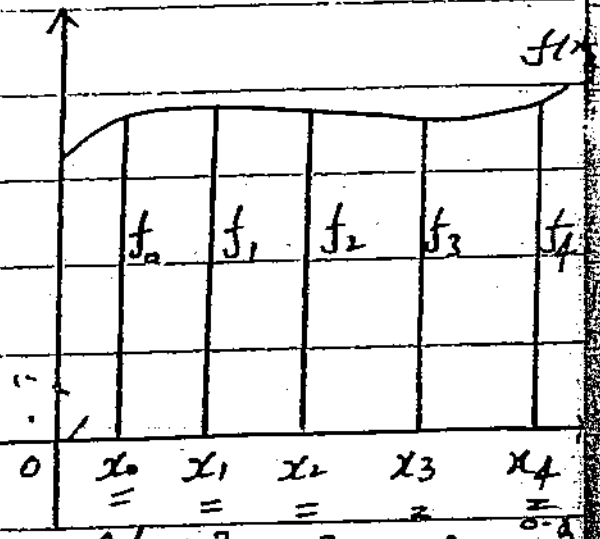
PAPER A/2014

Q6 Evaluate $\int_{0.1}^{0.5} \sin\theta \cdot d\theta$ using 5 Point Boole's Rule and Simson's Rule. Indicate which method is superior.

Solution:-

$n = 4$

$h = \frac{b-a}{n} = \frac{0.5-0.1}{4} = 0.1$



$f(x) = \sin\theta$

$f_0 = f(x_0)$

$= f(0.1) = \sin(0.1)$

$= 0.09983$

$f_1 = \sin(0.2)$

$= 0.1986$

$f_2 = \sin(0.3)$

$= 0.2955$

$f_3 = \sin(0.4) = 0.3894$

$f_4 = \sin(0.5) = 0.4794$

Solved by Labeeb Ahmad

NOTE Using Trignometric function Calculator

Q14

Boole's Rule :-

Using 5 point

$$\int_a^b f(x)\cdot dx = \frac{2h}{45}\left[7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4\right]$$

Indicate

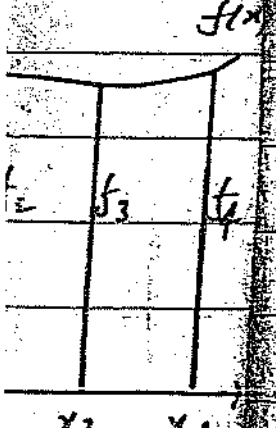
$$= \frac{2}{45}(0.1)\left[7(0.09983) + 32(0.1986)\right.$$

$$\left. + 12(0.2955) + 32(0.3894) + 7(0.4794)\right]$$

$$= (0.00444)\left[0.69881 + 6.3552 + 3.546 + 12.4608 + 3.3558\right]$$

$$= (0.00444)(26.41661)$$

$$= 0.1172897$$



Simpson 1/3 Rule :-

$$\int_{0.1}^{0.5} \sin\theta\cdot d\theta = \frac{h}{3}\left[(f_0 + f_4) + 4(f_1 + f_3) + 2f_2\right.$$

$$= \frac{(0.1)}{3}\left[(0.09983 + 0.4794) + 4(0.1986 + 0.3894) + 2(0.2955)\right]$$

$$(0.0333)\left[(0.57923) + 2.352 + 0.591\right]$$

$$= (0.0333)(3.52223)$$

calculat

$$= 0.117290$$

Solved by
Labeeb Ahmad

lved by
b Ahmad

Simpson 3/8 Rule

$$\int_{0.1}^{0.5} \sin\theta \cdot d\theta = \frac{3h}{8}\left[(f_0+f_4)+3(f_1+f_2)+2f_3\right]$$

$$= \frac{3(0.1)}{8}\left[(0.0998+0.4794)+3(0.1986+0.2955)+2(0.3894)\right]$$

$$= (0.0375)\left[(0.57923)+1.4823+0.7788\right]$$

$$= (0.0375)(2.84033) = 0.106512375$$

Exact value:- $\int_{0.1}^{0.5} \sin\theta \cdot d\theta = -\cos\theta\Big|_{0.1}^{0.5}$

$-\cos(0.5)+\cos(0.1) = 0.117421$

Hence Simpson 1/3 Rule is Superior.

PAPER A/2013

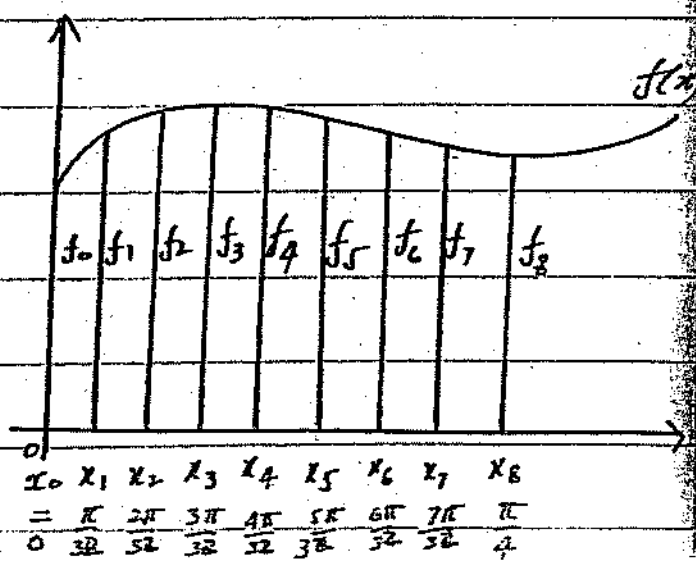
Q8(a) Apply 9 Points Trapezoidal Rule to evaluate $\int_0^{\pi/4} \sin x \cdot dx$

Solution

$n = 8$

$h = \frac{b-a}{n}$

$h = \frac{\frac{\pi}{4}-0}{8} = \frac{\pi}{32}$



Solved by

f(x

$f_0 = f$

$f_1 = f_1$

$f_2 = f($

$f_3 =$

$f_4 =$

$f_5 =$

$f_6 =$

$f_7 =$

$f_8 =$

Trape

$\int_0^{\pi/4}$ Si

$= \frac{\pi/3}{2}$

$f(x_i) = (0.0$

$= (0.$

$= 0.$

$f(x) = \sin x$

$f_0 = f(x_0) = f(0) = \sin(0) = 0$

$f_1 = f(x_1) = f(\pi/32) = \sin(\pi/32) = 0.098017$

$f_2 = f(x_2) = f(2\pi/32) = \sin(2\pi/32) = 0.1950$

$f_3 = \sin(3\pi/32) = 0.2902$

$f_4 = \sin(4\pi/32) = 0.3826$

$f_5 = \sin(5\pi/32) = 0.4713$

$f_6 = \sin(6\pi/32) = 0.5555$

$f_7 = \sin(7\pi/32) = 0.6343$

$f_8 = \sin(\pi/4) = 0.7071$

* Trapezoidal Rule

$$\int_0^{\pi/4} \sin x \cdot dx = \frac{h}{2}\left[(f_0 + f_8) + 2(f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7)\right]$$

$$= \frac{\pi/32}{2}\left[(0 + 0.7071) + 2(0.098017 + 0.1950 + 0.2902 + 0.3826 + 0.4713 + 0.5555 + 0.6343)\right]$$

$$= (0.049087)\left[(0.7071) + 2(2.626917)\right]$$

$$= (0.049087)(5.960934)$$

$$= 0.292604$$

Solved by

A/2013

Q9(b) Apply Weddles Rule evaluate $I_q =$

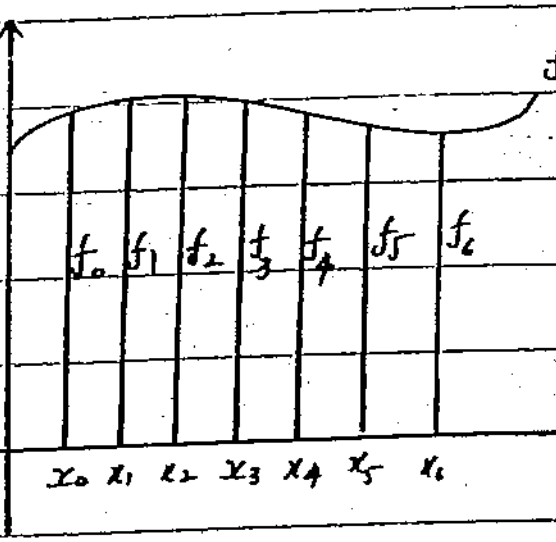
$$\int_{0.3}^{0.7} \sqrt{e^x + x^2 + 5}\, dx$$

Solution

$n = 6$

$$h = \frac{b-a}{n} = \frac{0.7-0.3}{6} = 0.0666$$

$$f(x) = \sqrt{e^x + x^2 + 5}$$



$f_0 = f(x_0)$

$= \sqrt{e^{0.3} + (0.3)^2 + 5}$

$f_0 = 2.5376$

$f_1 = f(x_0 + h) = f(0.3 + 0.0666) = f(0.3666)$

$f_1 = \sqrt{e^{0.3666} + (0.3666)^2 + 5}$

$= 2.5646$

$f_2 = f(x_0 + 2h) = f(0.4332)$

$f_2 = \sqrt{e^{0.4332} + (0.4332)^2 + 5}$

$f_2 = 2.5941$

$f_3 = f(x_0 + 3h) = f(0.4998)$

$= \sqrt{e^{(0.4998)} + (0.4998)^2 + 5} = 2.6264$

Solved by
Labeeb Ahmad

evaluate $f_4 = f(x_0 + 4h) = f(0.5664)$

$$= \sqrt{e^{0.5664} + (0.5664)^2 + 5}$$

$$= 2.66133$$

$f_5 = f(x_0 + 5h) = f(0.633)$

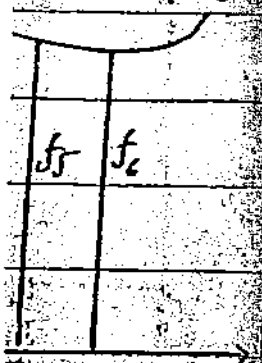
$$= \sqrt{e^{0.633} + (0.633)^2 + 5}$$

$$= 2.6988$$

$f_6 = f(x_0 + 6h) = f(0.6996)$

$$= \sqrt{e^{0.6996} + (0.6996)^2 + 5}$$

$f_6 = 2.7390$



(0.3666)

Weddle's Rule

$$\int_{0.3}^{0.7} \sqrt{e^x + x^2 + 5} = \frac{3h}{10}\left[f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + \right.$$

$$\frac{3(0.0666)}{10}\Big[2.5376 + 5(2.5646) + 2.5941$$

$$+ 6(2.6264) + 2.6613 + 5(2.6988$$

$$+ 2.7390\Big]$$

$$= (0.01998)\Big[2.5376 + 12.823 + 2.5941 +$$

$$15.7584 + 2.6613 + 13.494 + 2.7390$$

$$= (0.01998)(52.60743)$$

$$= 1.051096$$

ed by
) Ahmad

64

Solved by
Labeeb Ahmad

PAPER A/2012

Q6(a) Approximate the length of the following function by Simpson 1/3 Rule by taking 10 subintervals.

$$f(x) = e^{-x} \qquad 0 < x \leq 1$$

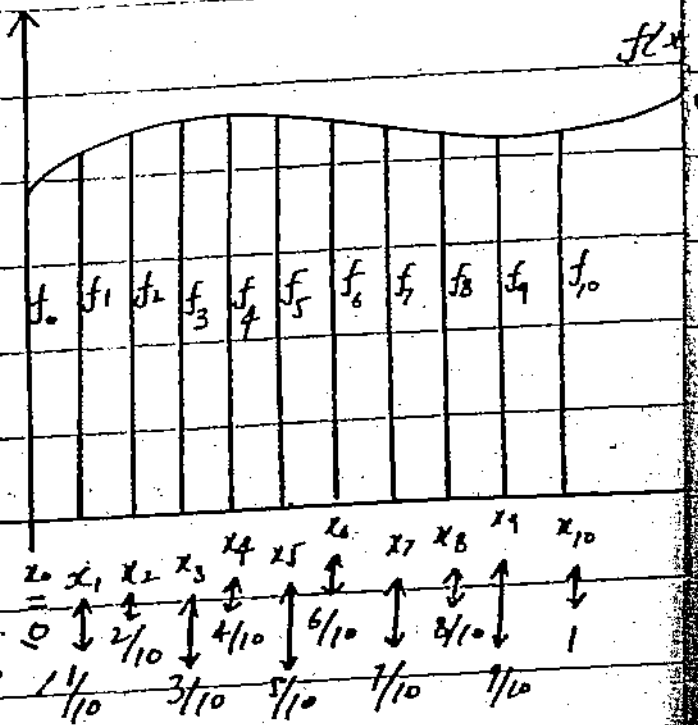
Solution:

$n = 10$

$h = \frac{b-a}{n} = \frac{1-0}{10}$

$h = \frac{1}{10}$

$f(x) = e^{-x}$



$f_0 = f(x_0) = e^{-0} = 1$

$f_1 = f(x_1) = e^{-1/10} = 0.9048$

$f_2 = e^{-2/10} = 0.8187$

$f_3 = e^{-3/10} = 0.7408$

$f_4 = e^{-4/10} = 0.6703$

$f_5 = e^{-5/10} = 0.6065$

$f_6 = e^{-6/10} = 0.5488$

$f_7 = e^{-7/10} = 0.4965$

$f_8 = e^{-8/10} = 1.0.4$

Solved by Labeeb Ahmad

2012

$f_9 = f(x_9) = e^{-9/10} = 0.4065$

$f_{10} = f(x_{10}) = e^{-1} = 0.3678$

th of the

le by

$x \le 1$

Simpson 1/3 Formula

$$\int_{x_0}^{x_0+10h} f(x)\cdot dx = \frac{h}{3}\left[(f_0+f_{10}) + 4(f_1+f_3+f_5+f_7+f_9) + 2(f_2+f_4+f_6+f_8)\right]$$

$$\int_0^1 e^{-x}dx = \frac{1}{10\times 3}\Big[(1+0.3678) + 4(0.9048+0.7408 + 0.6065+0.4965+0.4065) + 2(0.8187+0.6703 + 0.5488+0.4493)\Big]$$

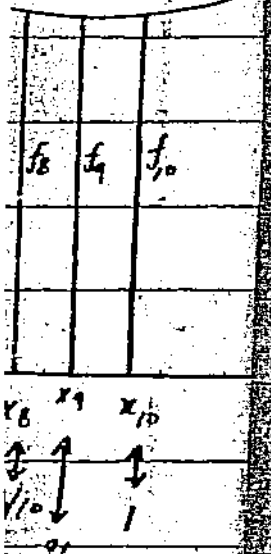
$$= (0.0333)\Big[(1.3678) + 4(3.1551) + 2(2.4871)\Big]$$

$$= (0.0333)\Big[(1.3678) + (12.6204) + (4.9742)\Big]$$

$$= (0.0333)(18.9624)$$

$$= 0.63144 \quad \text{Ans}$$

Solved by Labeeb Ahmad



2012

Q5(a) Evaluate $\int_{-1}^{1} \frac{dx}{x+2}$ by two point Gauss Quadrature formula.

Solution By Gauss Quadrature formula

$$I = \frac{b-a}{2}\left[F(1/\sqrt{3}) + F(-1/\sqrt{3})\right]$$

$$I = \frac{1+1}{2}\left[\frac{1}{1/\sqrt{3}+2} + \frac{1}{(-1/\sqrt{3})+2}\right] = \left[\frac{1}{2.5773} + \frac{1}{1.4226}\right]$$

$$= 0.3879 + 0.7029 = 1.09081$$

by

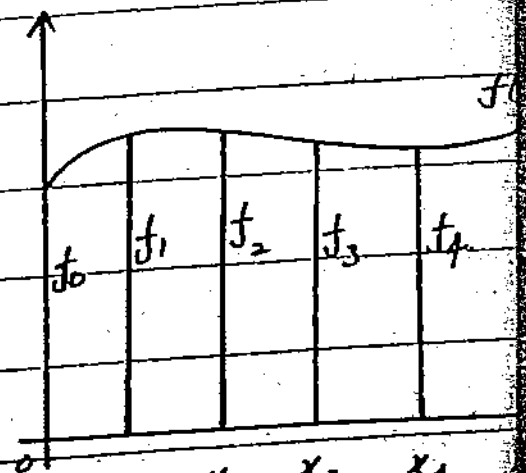
mad

PAPER A/2011

Q8(b) Solve the integral $\int_0^5 \frac{dx}{1+x^2}$ using (i) Simpson 1/3 rule (ii) Boole's formula. Also compare with exact value.

Solution:- Taking $h=1$, divide the whole range of the integration $[0,5]$ into 5 equal parts

$$f(x) = \frac{1}{1+x^2}$$



$$f_0 = f(x_0) = f(0) = \frac{1}{1+0} = 1$$

$$f_1 = \frac{1}{1+1^2} = 0.5$$

$$f_2 = \frac{1}{1+2^2} = 0.2$$

$$f_3 = \frac{1}{1+3^2} = 0.1$$

$$f_4 = \frac{1}{1+4^2} = 0.0588235$$

$$f_5 = \frac{1}{1+5^2} = 0.0384615$$

Solved by Labeeb Ahmad

2011

$\int_0^5 \frac{dx}{1+x^2}$

value.

vide the

the integration

x_5 = 3 x_4 = 4

Solved by Ahmad

Simpson 1/3 rule :-

$$\int_{x_0}^{x_0+5h} f(x)\cdot dx = \frac{h}{3}\left[(f_0+f_5)+4(f_1+f_3)+2(f_2+f_4)\right.$$

$$\int_0^5 \frac{dx}{1+x^2} = \frac{1}{3}\Big[(1+0.0384615)+4(0.5+0.1)+2(0.2+0.0588235$$

$$= \frac{1}{3}\Big[1.0384615+4(0.6)+2(0.25882$$

$$= \frac{1}{3}\Big[1.0384615+2.4+0.517647\Big]$$

$$= \frac{1}{3}(3.9561085)$$

$$= 1.31870283$$

Boole's rule

$$\int_{x_0}^{x_0+5h} f(x)\cdot dx = \frac{2h}{45}\Big[7(f_0+2f_4)+32(f_1+f_3+f_5)+12(f_2)\Big]$$

$$\int_0^5 \frac{dx}{1+x^2} = \frac{2}{45}\Big[7(1+2(0.0588235))+32(0.5+0.1+0.0384615)+12(0.2)\Big]$$

Solved by Lahore

$$= \frac{2}{45}\left[7(1.117647) + 32(0.6384615) + 12(0.2\right.$$

$$= \frac{2}{45}\left[7.823529 + 20.430768 + 2.4\right]$$

$$= \frac{2}{45}\left[30.654297\right]$$

$$= 1.3624132$$

Now $\int_0^5 \frac{1}{1+x^2}\,dx = \tan^{-1}x\Big|_0^5$

$$= \tan^{-1}(5) - \tan^{-1}(0)$$

$$= 1.373400767 - 0$$

$$= 1.373400767$$

Which shows that the value of the integral found by Boole's rule is the nearest to the actual value than others.

Written by
Labeeb Ahmad
Mob:0341-7224005

Q8(

(i)

(ii)

Also

Solu

$f(x) =$

$f_0 = f$

$f_0 = f($

$= 3.0$

$f_1 = f($

$= \sin$

$= 2.7$

PAPER S/2011

Q8(b) Calculate the value of following integral

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$$ by

(i) Simpson 3/8 Rule

(ii) Weddle's rule

Also compare the error with exact value.

Solution:-

Take $n = 6$

$$h = \frac{b-a}{n} = \frac{1.4-0.2}{6} = 0.2$$

$f(x) = \sin x - \ln x + e^x$

$f_0 = f(x_0)$

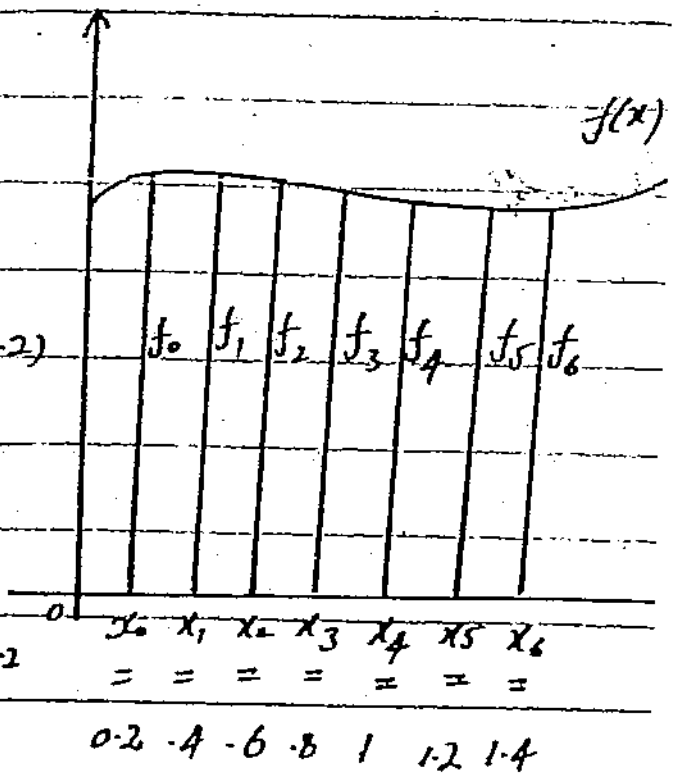
$f_0 = f(0.2) = \sin(0.2) - \ln(.2) + e^{.2}$

$= 3.0295$

$f_1 = f(0.4)$

$= \sin(0.4) - \ln(0.4) + e^{0.2}$

$= 2.7975$



Solved by

$f_2 = \sin(0.6) - \ln(0.6) + e^{0.6} = 2.8975$

$f_3 = \sin(0.8) - \ln(0.8) + e^{0.8} = 3.1660$

$f_4 = \sin(1.0) - \ln(1) + e^{1} = 3.5597$

$f_5 = \sin(1.2) - \ln(1.2) + e^{1.2} = 4.0698$

$f_6 = \sin(1.4) - \ln(1.4) + e^{1.4} = 4.70417$

Simpson 3/8 rule

$$\int_{x_0}^{x_0+6h} f(x)\,dx = \frac{3h}{8}\left[(f_0 + f_6) + 3(f_1 + f_2 + f_4 + f_5) + 2f_3\right]$$

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\,dx = \frac{3}{8}(0.2)\Big[(3.0295 + 4.70417) + 3(2.7975 + 2.8975 + 3.5597 + 4.0698) + 2(3.1660)\Big]$$

$$= (0.075)\Big[(7.7336) + 3(13.3245) + 6.332\Big]$$

$$= (0.975)(54.0391)$$

$$= 4.0529325$$

Solved by
Labeeb Ahmad

11

Weddle's Rule

$$\int_{x_0}^{x_0+6h} f(x)\cdot dx = \frac{3h}{10}\left[f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5\right.$$

$$\int_{0.2}^{1.4} \sin x - \ln x + e^x = \frac{3(0.2)}{10}\left[3.0295 + 5(2.7975 + 2.8975 + 6(3.1660) + 3.5597 + 5(4.0698 + 4.70417)\right]$$

$$= (0.06)\left[(3.0295) + 13.9875 + 2.8975 + 18.996 + 3.5597 + 20.349 + 4.70417\right]$$

$$= (0.06)[67.52337]$$

$$= 4.0514022$$

Exact value

$$\int_{0.2}^{1.4} (\sin x - \ln x + e^x)\,dx$$

$$= -\cos x + (x - x\ln x) + e^x \Big|_{0.2}^{1.4}$$

$$= [-\cos(1.4) + \cos(0.2)] + [1.4 - (1.4)\ln(1.4) - (0.2) + (0.2)\ln(0.2)] + [e^{1.4} - e^{0.2}]$$

$$= (0.81009) + (0.407050) + (2.8337)$$

$$= 4.0508494$$

Simpson 3/8 rule	Exact value	Error = | True − Approximate |
4.052932	4.050849	Error = 0.002083
Weddle rule		
4.0514022	4.050849	0.00055532

PAPER S/2009

Q6 {b} Evaluate $\int_0^{\pi/2} \sin x \cdot dx$ using 4 points

(i) Simpson 1/3 Rule

(ii) Trapezodial Rule

and Compare with exact solution

Solution:-

Here $n = 4$

$$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}$$

$f(x) = \sin x$

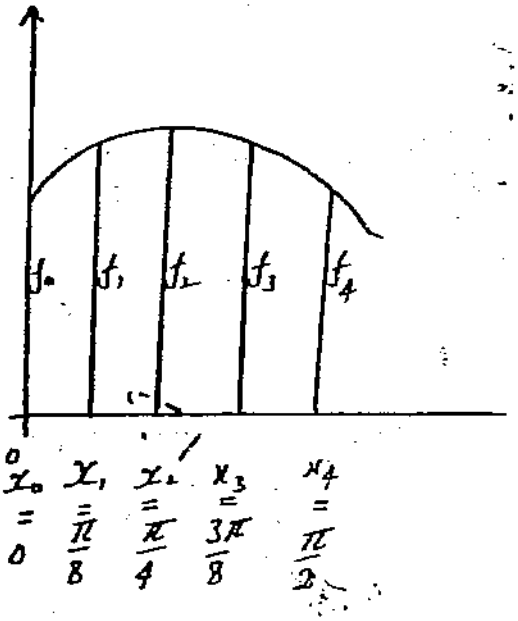
$f_0 = f(x_0) = \sin(0) = 0$

$f_1 = f(x_0 + h) = \sin \frac{\pi}{8} = 0.3826$

$f_2 = f(x_0 + 2h) = \sin \frac{2\pi}{8} = 0.7071$

$f_3 = f(x_0 + 3h) = \sin \frac{3\pi}{8} = 0.9238$

$f_4 = f(x_0 + 4h) = \sin \frac{\pi}{2} = 1$



Simpson's 1/3 Rule

$$\int_{x_0}^{x_0+4h} f(x) \cdot dx = \frac{h}{3}\left[(f_0 + f_4) + 4(f_1 + f_3) + 2(f_2)\right]$$

$$= \frac{\pi/8}{3}\left[(0+1) + 4(0.3826 + 0.9238) + 2(0.7071)\right]$$

$$= (0.1308)\left[1 + 5.2256 + 1.4142\right]$$

$$= 0.9999$$

Solved by
Labeeb Ahmad

F

Q6 a

5 point

(i)

(ii)

and

Solu

$f(x)$

$f_0 =$

$f_1 = f($

$f_2 = f($

$f_3 = f(x_0$

$f_4 = f($

Simp.

$$\int_{x_0}^{x_0+4}$$

Solve

Jabeeb

$= \left[-\cos\frac{\pi}{2} + \cos(0)\right]$

$= (0 + 1)$

$= 1$

		Exact value	Error
Simpson's 1/3 Rule	0.99914	1	0.00086
Simpson's 3/8 Rule	0.575249	1	0.42477
Trapezoidal Rule	0.99665	1	0.00335

Hence Simpson's $\frac{1}{3}$ rule has very small error. Therefore answer of Simpson's 1/3 rule is more accurate than other.

Solved by
Labeeb Ahmad
0341-7224005

$= (0.07363)[1 + 3(1.2847) + 2(1.4793)$

$= (0.07363)(1 + 3.8541 + 2.9586)$

$= (0.07363)(7.8127)$

$= 0.575249$

Trapezoidal Rule

$$\int_{x_0}^{x_0+8h} f(x)\,dx = \frac{h}{2}\left[(f_0 + f_8) + 2(f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7)\right]$$

$$= \frac{\pi/16}{2}\left[(0+1) + 2(0.19509 + 0.3826 + 0.5555 + 0.2071 + 0.8314 + 0.9238 + 0.9807)\right]$$

$= (0.09817)[1 + 2(4.57619)]$

$= (0.09817)(10.15238)$

$= 0.996659$

Exact value

Solved by Labeeb Ahmad

$$\int_0^{\pi/2} \sin x \cdot dx$$

$$= -\cos x \Big|_0^{\pi/2}$$

Simpson's 1/3 Rule

using
Point
from for

$$\int_{x_0}^{x_0+8h} f(x)\,dx = \frac{h}{3}\left[(f_0+f_8)+4(f_1+f_3+f_5+f_7)+2(f_2+f_4+f_6)\right]$$

$$\int \sin x\,dx = \frac{\pi/16}{3}\left[(0+1)+4(0.1950+0.5555+0.8314+0.9807)+2(0.3826+0.7071+0.9238)\right]$$

$$= \frac{\pi}{16}$$

$$= (0.0654)\left[1+4(2.5626)+2(2.0135)\right]$$

$$= (0.0654)\left[1+10.2504+4.027\right]$$

$$= (0.0654)(15.2774)$$

$$= 0.99914196$$

Solved by
Labeeb Ahmad

Simpson's 3/8 Rule

$$\int_{x_0}^{x_0+8h} f(x)\,dx = \frac{3h}{8}\left[(f_0+f_8)+3(f_1+f_2+f_4+f_5+f_7)+2(f_3+f_6)\right]$$

$$\int_0^{\pi/2} \sin x\,dx = \frac{3\times(\pi/16)}{8}\left[(0+1)+3(0.1950+0.3826+0.7071+0.8314+0.9807)+2(0.5555+0.9238)\right]$$

PAPER A/2010

Q6{b} Evaluate $\int_0^{\pi/2} \sin x \cdot dx$ using 9 point Simpson's Rule and 9 point Trapezodial Rule also compare them for accuracy

Solution:- $n = 8$

$$h = \frac{b-a}{8} = \frac{\pi/2 - 0}{8} = \frac{\pi}{16}$$

$f(x) = \sin x$

$f_0 = f(x_0) = \sin(0) = 0$

$f_1 = f(x_0 + h) = \sin\left(\frac{\pi}{16}\right) = 0.19509$

$f_2 = f(x_0 + 2h) = \sin\left(\frac{2\pi}{16}\right) = 0.38268$

$f_3 = f(x_0 + 3h) = \sin\left(\frac{3\pi}{16}\right) = 0.5555$

$f_4 = f(x_0 + 4h) = \sin(4\pi/16) = 0.7071$

$f_5 = f(x_0 + 5h) = \sin(5\pi/16) = 0.8314$

$f_6 = f(x_0 + 6h) = \sin(6\pi/16) = 0.9238$

$f_7 = f(x_0 + 7h) = \sin(7\pi/16) = 0.9807$

$f_8 = f(x_0 + 8h) = \sin(\pi/2) = 1$

Solved by

PAPER A/2015

Q8 {a} Find the first and 2nd derivatives of the function tabulated at the point $x = 1.1$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	1.296	2.432	4

Solution :- Solved by Labeeb Ahmad

x	$f(x) = y$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	0					
		0.128				
1.2	0.128		0.288			
		0.416		0.048		
1.4	0.544		0.336		0	
		0.752		0.048		0
1.6	1.296		0.384		0	
		1.136		0.048		
1.8	2.432		0.432			
		1.568				
2.0	4					

$$h = 1.2 - 1.0 = 0.2$$

$$\text{And } r = \frac{x - x_0}{h} = \frac{1.1 - 1.0}{0.2} = 0.5$$

Trapezoidal Rule

$$\int_{x_0}^{x_0+4h} f(x)\,dx = \frac{h}{2}\left[(f_0+f_4)+2(f_1+f_2+f_3)\right]$$

$$= \frac{\pi/8}{2}\left[(0+1)+2(0.3826+0.7071+0.9238)\right]$$

$$= (0.1963)\left[1+2(2.0135)\right]$$

$$= (0.1963)(5.027)$$

$$= 0.9868001$$

Exact value:-

$$\int_0^{\pi/2} \sin x\,dx$$

$$= -\cos x \Big|_0^{\frac{\pi}{2}}$$

$$= -\cos\frac{\pi}{2} + \cos(0)$$

$$= 0 + 1$$

$$= 1$$

Hence the value of Simpson 1/3 Rule more accurate than Trapezoidal Rule.

Solved by
Labeeb Ahmad

PAPER A/2014

Q9(a) Find the first and second derivatives of the function tabulated below at the point $x = 1.8$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	1.296	2.432	4

Solution:-

Solved by
Labeeb Ahmad

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1.0	0	→ 0.128				
			→ 0.288			
1.2	0.128	→ 0.416		→ 0.048		
			→ 0.336		→ 0	
1.4	0.544	→ 0.752		→ 0.048		→ 0
			→ 0.384		→ 0	
1.6	1.296	→ 0.136		→ 0.048		
			→ 0.432			
1.8	2.432	→ 1.568				
2.0	4					

$$h = 1.2 - 1.0 = 0.2$$

$$\sigma = \frac{x - x_n}{h} = \frac{1.8 - 2.0}{0.2} = -1$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2r-1)}{2}\Delta^2 y_0 + \frac{(3r^2-6r+2)}{6}\Delta^3 y_0\right]$$

$$\frac{dy}{dx} = \frac{1}{0.2}\left[(0.128) + \frac{[2(0.5)-1]}{2}\cdot(0.288) + \frac{[3(0.5)^2-6(0.5)+2]}{6}\right.$$
$$\left.(0.048)\right]$$

$$\frac{dy}{dx} = \frac{1}{0.2}\left[(0.128) + 0 - 0.002\right]$$

$$\frac{dy}{dx} = 0.63$$

Now Second derivative

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6r-6}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{(0.2)^2}\left[(0.288) + \frac{(6(0.5)-6)}{3!}\cdot(0.048)\right]$$

$$= \frac{1}{0.04}\left[(0.288) + (-0.024)\right]$$

$$= \frac{1}{0.04}(0.264)$$

$$\frac{d^2y}{dx^2} = 6.6$$

Solved by
Labeeb Ahmad

PAPER A/2014

Q3(b) $\mu^2 = 1 + \frac{\delta^2}{4}$

Take R.H.S $\Rightarrow 1 + \frac{\delta^2}{4}$

As we know that $\delta = E^{1/2} - E^{-1/2}$

$$\Rightarrow 1 + \frac{(E^{1/2} - E^{-1/2})^2}{4}$$

$$= \frac{4 + [E + E^{-1} - 2]}{4}$$

$$= \frac{E + E^{-1} + 2}{4} = \frac{(E^{1/2})^2 + (E^{-1/2})^2 + 2 \cdot E^{1/2} \cdot 1}{4}$$

$$= \frac{[E^{1/2} + E^{-1/2}]^2}{4} = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right]$$

$$= \mu^2$$

(ii)

$$\Delta\left(\frac{f_n}{g_n}\right) = \frac{g_n \Delta f_n - f_n \Delta g_n}{g_n \cdot g_{n+1}}$$

Take L.H.S.

$$\Delta\left(\frac{f_n}{g_n}\right) = \Delta(h_n) \qquad \therefore \left(h_n = \frac{f_n}{g_n}\right)$$

$$= h_{n+1} - h_n$$

Solved by
Labeeb Ahmad

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2r+1}{2!}\nabla^2 y_n + \frac{3r^2+6r+2}{3!}\nabla^3 y_n\right]$$

$$\frac{dy}{dx} = \frac{1}{0.2}\left[1.568 + \frac{2(-1)+1}{2}(0.432) + \frac{\{3(-1)^2+6(-1)+2\}(0.048)}{6}\right]$$

$$= \frac{1}{0.2}\left[1.568 + (-0.5)(0.432) + (-0.1666)(0.048)\right]$$

$$= \frac{1}{0.2}\left[1.568 - 0.216 - 0.008\right]$$

$$\frac{dy}{dx} = 6.72$$

And Second Derivative

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6r+6}{6}\nabla^3 y_n\right]$$

$$= \frac{1}{(0.2)^2}\left[0.432 + \frac{[6(-1)+6](0.048)}{6}\right]$$

$$= \frac{1}{0.04}\left[0.432 + 0\right]$$

$$\frac{d^2y}{dx^2} = 10.8$$

Solved by
Labeeb Ahmad

$$\left[1+\frac{1}{2}\delta^2\right]^2 = \left[1+\frac{1}{2}\left[E^{1/2}-E^{-1/2}\right]^2\right]^2$$

$$= \left[1+\frac{1}{2}\left(E+E^{-1}-2\right)\right]^2$$

(i)

$$= \left[\frac{2+E+E^{-1}-2}{2}\right]^2$$

$$= \left[\frac{E+E^{-1}}{2}\right]^2$$

$$= \frac{4+\left(E-E^{-1}\right)^2}{4}$$

(ii)

$$= 1+\left[\frac{E-E^{-1}}{2}\right]^2$$

$$= 1+\left[\left(\frac{E^{1/2}+E^{-1/2}}{2}\right)\left(E^{1/2}-E^{-1/2}\right)\right]^2$$

$$= 1+\left(\mu\ \delta\right)^2$$

$$= 1+\mu^2\delta^2 \quad = \text{L.H.S.}$$

(iv)

$$\mu = \frac{2+\Delta}{2\sqrt{1+\Delta}} = \frac{2-\nabla}{2\sqrt{1-\nabla}}$$

Take $\dfrac{2+\Delta}{2\sqrt{1+\Delta}} = \dfrac{1+1+\Delta}{2\sqrt{1+\Delta}}$

As we know that $(1+\Delta = E)$

Solved by Labeeb Ahmad

ved by
b Ahmad

$$= \frac{f_{n+1}}{g_{n+1}} - \frac{f_n}{g_n}$$

$$= \frac{g_n f_{n+1} - f_n g_{n+1}}{g_n g_{n+1}} \quad \text{(i)}$$

we also note that

$$\Delta f_n = f_{n+1} - f_n$$

$$\Delta f_n - f_n = f_{n+1} \quad \text{(ii)}$$

Similarly $\Delta g_n - g_n = g_{n+1}$

put this value in (i)

$$= \frac{g_n[\Delta f_n - f_n] - f_n[\Delta g_n - g_n]}{g_n \cdot g_{n+1}}$$

$$= \frac{g_n \Delta f_n - f_n g_n - f_n \Delta g_n + f_n g_n}{g_n \cdot g_{n+1}}$$

$$= \frac{g_n \Delta f_n - f_n \Delta g_n}{g_n \cdot g_{n+1}}$$

$= R.H.S.$

Solved by
Labeeb Ahmad

(iii)

$$1 + \mu^2 \delta^2 = \left(1 + \frac{\delta^2}{2}\right)^2$$

Take R.H.S. $\left(1 + \frac{\delta^2}{2}\right)^2$

PAPER A/2013

Q1(b)

(i)

$$\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}}$$

Solution :-

Take R.H.S $= \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}}$

$$= \frac{\delta^2}{2} + \delta\sqrt{\frac{4+\delta^2}{4}}$$

$$= \frac{\delta^2}{2} + \frac{\delta}{2}\sqrt{4+\delta^2}$$

$$= \frac{\delta}{2}\left[\delta + \sqrt{4+\delta^2}\right]$$

$$= \frac{\delta}{2}\left[(E^{\frac{1}{2}} - E^{-\frac{1}{2}}) + \sqrt{4+(E^{1/2}-E^{-1/2})^2}\right.$$

$$= \frac{\delta}{2}\left[(E^{1/2} - E^{-1/2}) + \sqrt{4+(E^{\frac{1}{2}})^2+(E^{-\frac{1}{2}})^2-2}\right.$$

$$= \frac{\delta}{2}\left[(E^{1/2} - E^{-1/2}) + \sqrt{(E^{1/2})^2+(E^{-1/2})^2+2}\right.$$

$$= \frac{\delta}{2}\left[(E^{1/2} - E^{-1/2}) + \sqrt{(E^{1/2}+E^{-1/2})^2}\right]$$

$$= \frac{\delta}{2}\left[E^{1/2} - E^{-1/2} + E^{1/2} + E^{-1/2}\right]$$

$$= \frac{1}{2}(E^{1/2} - E^{-1/2})(2E^{1/2})$$

$$= E - 1$$

$$= \Delta = L.H.S$$

Solved by
Labeeb Ahmad

$$= \frac{1+E}{2\sqrt{E}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{E}} + \frac{E}{\sqrt{E}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{E}} + \sqrt{E}\right] = \frac{1}{2}\left[E^{-1/2} + E^{+1/2}\right]$$

$$= \mu \quad \text{——— (i)}$$

Now take

$$\frac{2-\nabla}{2\sqrt{1-\nabla}} = \frac{1+1-\nabla}{2\sqrt{1-\nabla}}$$

As we know that $1-\nabla = E^{-1}$

$$= \frac{1+E^{-1}}{2\sqrt{E^{-1}}} = \frac{1}{2}\left[\frac{1}{\sqrt{E^{-1}}} + \frac{E^{-1}}{\sqrt{E^{-1}}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{E^{-1}}} + \sqrt{E^{-1}}\right]$$

Solved by
Labeeb Ahmad

$$= \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$$

$$= \mu \quad \text{——— (ii)}$$

From (i) & (ii)

$$\mu = \frac{2+\Delta}{2\sqrt{1+\Delta}} = \frac{2-\nabla}{2\sqrt{1-\nabla}}$$

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PAPER A/2012

Q4(a) $\mu(f_k g_k) = \mu f_k \cdot \mu g_k + \frac{1}{4}\delta f_k \cdot \delta g_k$

Solution

Take R.H.S

$$\mu f_k \cdot \mu g_k + \frac{1}{4}\delta f_k \cdot \delta g_k \quad \text{---(i)}$$

As
$$\left.\begin{aligned} \mu f_n &= \frac{1}{2}\left[f_{n+\frac{h}{2}} + f_{n-\frac{h}{2}}\right] \\ \delta f_n &= \left[f_{n+\frac{h}{2}} - f_{n-\frac{h}{2}}\right] \end{aligned}\right\} \quad \text{---(ii)}$$

$E^{-1/2}$)] Using these Definitions in (i)

$=E^{-1/2}$] $\frac{1}{2}\left[f_{k+\frac{h}{2}} + f_{k-\frac{h}{2}}\right]\cdot\frac{1}{2}\left[g_{k+\frac{h}{2}} + g_{k-\frac{h}{2}}\right] +$

Solved by Labeeb Ahmad $\frac{1}{4}\left[f_{k+\frac{h}{2}} - f_{k-\frac{h}{2}}\right]\left[g_{k+\frac{h}{2}} - g_{k-\frac{h}{2}}\right]$

$$\frac{1}{4}\left[f_{k+\frac{h}{2}}\cdot g_{k+\frac{h}{2}} + f_{k+\frac{h}{2}}\cdot g_{k-\frac{h}{2}} + f_{k-\frac{h}{2}}\cdot g_{k+\frac{h}{2}} + f_{k-\frac{h}{2}}\cdot g_{k-\frac{h}{2}}\right]$$

$$\frac{1}{4}\left[f_{k+\frac{h}{2}}\cdot g_{k+\frac{h}{2}} - f_{k+\frac{h}{2}} g_{k-\frac{h}{2}} - f_{k-\frac{h}{2}}\cdot g_{k+\frac{h}{2}} + f_{k-\frac{h}{2}}\cdot g_{k-\frac{h}{2}}\right]$$

$$\frac{1}{4}\left[2\left(f_{k+\frac{h}{2}}\cdot g_{k+\frac{h}{2}} + f_{k-\frac{h}{2}}\cdot g_{k-\frac{h}{2}}\right)\right]$$

$$\frac{1}{2}\left[f_{k+\frac{h}{2}}\cdot g_{k+h} + f_{k-\frac{h}{2}}\cdot g_{k-h}\right] = \mu(f_k \cdot g_k)$$

(ii)

$\mu\delta = \frac{1}{2}(\Delta + \nabla)$

Take R.H.S $= \frac{1}{2}(\Delta + \nabla)$

As $\Delta = E - 1$

$\nabla = 1 - E^{-1}$

$= \frac{1}{2}\left[E - 1 + 1 - E^{-1}\right]$

$= \frac{1}{2}\left(E - E^{-1}\right)$

$= \frac{1}{2}\left[\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)\left(E^{1/2} - E^{-1/2}\right)\right]$

$= \left[\frac{1}{2}\left(E^{\frac{1}{2}} + E^{-1/2}\right)\right]\left[E^{1/2} - E^{-1/2}\right]$

$= \mu\delta$ L.H.S

(iii)

$\mu = \frac{2+\Delta}{2\sqrt{1+\Delta}}$

Solved by Labeeb Ahmad

Solution

see Paper A/2014 Q3(b)(iv)

Q.1

$$\nabla^2 = h^2D^2 + \frac{h^4D^4}{4} + \frac{h^6D^6}{36} + \dots\dots\dots$$

$$- h^3D^3 + \frac{h^4D^4}{3} + \dots\dots$$

$$\nabla^2 = h^2D^2 - h^3D^3 + \frac{h^4D^4}{4} + \frac{h^4D^4}{3} + \dots\dots$$

$$\nabla^2 = h^2D^2 - h^3D^3 + \frac{7}{12}h^4D^4 + \dots\dots$$

Hence proved

Solved by
Labeeb Ahmad

PAPER S|2021

Q4(a) $\mu\delta = \frac{1}{2}(\Delta + \nabla)$

Solution :- See Paper A/2013 Q1{b}(iii)

Q4(a(ii)) $\nabla^2 = h^2D^2 - h^3D^3 + \frac{7}{12}h^4D^4 + \cdots$

Solution :-

As we know

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Replace x by $-hD$

$$e^{-hD} = 1 - hD + \frac{h^2D^2}{2!} - \frac{h^3D^3}{3!} + \cdots$$

Also $e^{-hD} = 1 - \nabla$

$$1 - \nabla = 1 - hD + \frac{h^2D^2}{2} - \frac{h^3D^3}{6} + \cdots$$

$$\nabla = hD - \frac{h^2D^2}{2} + \frac{h^3D^3}{6} - \cdots$$

$$\nabla^2 = \left[hD - \frac{h^2D^2}{2} + \frac{h^3D^3}{6} - \cdots\right]^2 \quad \text{--- (i)}$$

As we know that

$$[a+b+c+\cdots]^2 = a^2+b^2+c^2+\cdots+2ab+2ac+2cd+\cdots$$

$$\nabla^2 = (hD)^2 + \left(\frac{h^2D^2}{2}\right)^2 + \left(\frac{h^3D^3}{6}\right)^2 + \cdots$$

$$+ 2(hD)\left(-\frac{h^2D^2}{2}\right) + 2\frac{(hD)(h^3D^3)}{6} + \cdots$$

$\nabla^2 = h^2\delta^2$

$\nabla^2 = h^2D^2$

$\nabla^2 = h^2D^2$

Solved by Labeeb Ahmad

$\Rightarrow 1 + \left(\frac{E - E^{-1}}{2}\right) + \left(\frac{E + E^{-1} - 1}{2}\right)$

$= 1 + \frac{E}{2} - \frac{E^{-1}}{2} + \frac{E}{2} + \frac{E^{-1}}{2} - 1$

$= \frac{2E}{2}$

$= E = L.H.S$

Solved by
Labeeb Ahmad

PAPER S1200

Q4(a) (i) $\Delta(f_n \cdot g_n) = f_{n+1}\Delta g_n + g_n \Delta f_n$

Take L.H.S

$\Delta(f_n \cdot g_n) = f_{n+1} \cdot g_{n+1} - f_n \cdot g_n$ —(i)

As $\Delta g_n = g_{n+1} - g_n$

$\Delta g_n + g_n = g_{n+1}$ —(ii)

Also $\Delta f_n = f_{n+1} - f_n$

$f_n = f_{n+1} - \Delta f_n$ —(iv)

put the value of (ii) & (iv) in (i)

$\Delta(f_n \cdot g_n) = f_{n+1}[\Delta g_n + g_n] - [f_{n+1} - \Delta f_n] g_n$

PAPER A/2009

Q5(a)

(i)

$$\mu^2 = 1 + (\delta^2/4)$$

Sol Take R.H.S $1 + \frac{\delta^2}{4}$

$$\Rightarrow 1 + \frac{[E^{1/2} - E^{-1/2}]^2}{4}$$

$$\Rightarrow \frac{4 + (E + E^{-1} - 2)}{4}$$

$$= \frac{E + E^{-1} + 2}{4}$$

$$= \frac{(E^{1/2})^2 + (E^{-1/2})^2 + 2 \cdot E^{1/2} \cdot E^{-1/2}}{(2)^2}$$

$$= \left[\frac{E^{1/2} + E^{-1/2}}{2}\right]^2$$

$$= \mu^2 = \text{L-H-S}$$

(ii)

$$E = 1 + \mu\delta + \frac{\delta^2}{2}$$

Solved by Labeeb Ahmad

Take R.H.S.

$$1 + \mu\delta + \frac{\delta^2}{2} = 1 + \left[\frac{1}{2}(E^{1/2} + E^{-1/2})\right]\left[E^{1/2} - E^{-1/2}\right] + \frac{[E^{1/2} - E^{-1/2}]^2}{2}$$

$$= 1 + \left[\frac{1}{2}\right]\left[E - E^{-1}\right] + \left[\frac{E + E^{-1} - 2}{2}\right]$$

PAPER A/2015

Q9

$$(E^2 - 5E + 6)y_n = n^2 - 1$$

Sol

$$E^2 y_n - 5E\,y_n + 6y_n = n^2 - 1$$

$$y_{n+2} - 5y_{n+1} + 6y_n = n^2 - 1$$

Let $y_n = Ab^n$

So the Difference Equation becomes

$$Ab^{n+2} - 5Ab^{n+1} + 6Ab^n = 0$$

$$Ab^n[b^2 - 5b + 6] = 0$$

Since $Ab^n \neq 0$ Hence $b^2 - 5b + 6 = 0$

$$b^2 - 2b - 3b + 6 = 0$$

$$b(b-2) - 3(b-2) = 0$$

$$(b-2)(b-3) = 0$$

$$b - 2 = 0 \quad \text{or} \quad b - 3 = 0$$

$$b = 2 \quad \text{or} \quad b = 3$$

Solved by
Labeeb Ahmad

So the Complementry solution is

$$y_c = C_1(2)^n + C_2(3)^n$$

Now we find Particular integral

Put $y_n = C_3 + C_4 n + C_5 n^2$

So the Difference Equation becomes

$$[C_3 + C_4(n+2) + C_5(n+2)^2] - 5[C_3 + C_4(n+1) + C_5(n+1)^2] + 6[C_3 + C_4 n + C_5 n^2] = n^2 - 1$$

$= f_{n+1}\Delta g_n + \cancel{f_{n+1} g_n} - \cancel{f_{n+1} g_n} + g_n \Delta f_n$

$= f_{n+1}\Delta g_n + g_n \Delta f_n$

$= R \cdot H \cdot S$ ✓ Hence Proved

(iii)

$\nabla - \Delta = \nabla\Delta$

Take R.H.S $\nabla\Delta f(x) = \nabla[\Delta f(x)]$

$= \nabla[f(x+h) - f(x)]$

$= \nabla f(x+h) - \nabla f(x)$

$= [f(x+h) - f(x)] - \nabla f(x)$

$= \Delta f(x) - \nabla f(x)$

$\nabla\Delta f(x) = (\Delta - \nabla) f(x)$

$\nabla\Delta = \Delta - \nabla = L \cdot H \cdot S$

Hence Proove

(iii)

$\Delta + \nabla = (\Delta/\nabla) - (\nabla/\Delta)$

Take R.H.S. $= \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$= \frac{\Delta^2 - \nabla^2}{\nabla\Delta} = \frac{(\Delta+\nabla)(\Delta-\nabla)}{\nabla\Delta}$

$= \frac{(\Delta+\nabla)(\Delta-\nabla)}{(\Delta-\nabla)}$

$= \Delta + \nabla = L \cdot H \cdot S$

Hence proved ✓

Solved by
Labeeb Ahmad

PA

Q9

Sol

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$C_3 + C_4$

$+ C_5 (n$

PAPER A/2015

Q9

$(E^2-5E+6)y_n = n^2-1$

Sol $E^2y_n - 5Ey_n + 6y_n = n^2-1$

$y_{n+2} - 5y_{n+1} + 6y_n = n^2-1$

Let $y_n = Ab^n$

So the Difference Equation becomes

$Ab^{n+2} - 5Ab^{n+1} + 6Ab^n = 0$

$Ab^n[b^2-5b+6] = 0$

Since $Ab^n \neq 0$ Hence $b^2-5b+6=0$

$b^2-2b-3b+6=0$

$b(b-2)-3(b-2)=0$

$(b-2)(b-3)=0$

$b-2=0$ or $b-3=0$

$b=2$ or $b=3$

So the Complementry solution is :-

$y_c = c_1(2)^n + c_2(3)^n$

Now we find Particular integral

Put $y_n = c_3' + c_4 n + c_5 n^2$

So the Difference Equation becomes

$[c_3 + c_4(n+2) + c_5(n+2)^2] - 5[c_3 + c_4(n+1) + c_5(n+1)^2] + 6[c_3 + c_4 n + c_5 n^2] = n^2-1$

Solved by
Labeeb Ahmad

$= f_{n+1}\cdot\Delta g_n + f_{n+1}\cdot g_n - f_{n+1}\cdot g_n + g_n\Delta f_n$

$= f_{n+1}\Delta g_n + g_n \Delta f_n$

$= R\cdot H\cdot S$ ✓ Hence Proved

(iii)

$\nabla - \Delta = \nabla\Delta$

Take R.H.S $\nabla\Delta f(x) = \nabla[\Delta f(x)]$

$= \nabla[f(x+h) - f(x)]$

$= \nabla f(x+h) - \nabla f(x)$

$= [f(x+h) - f(x)] - \nabla f(x)$

$= \Delta f(x) - \nabla f(x)$

$\nabla\Delta f(x) = (\Delta - \nabla) f(x)$

$\nabla\Delta = \Delta - \nabla = L\cdot H\cdot S$

Hence Proove

(iii)

$\Delta + \nabla = (\Delta/\nabla) - (\nabla/\Delta)$

Take R.H.S $= \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$= \frac{\Delta^2 - \nabla^2}{\nabla\Delta} = \frac{(\Delta+\nabla)(\Delta-\nabla)}{\nabla\Delta}$

$= \frac{(\Delta+\nabla)(\Delta-\nabla)}{(\Delta-\nabla)}$

$= \Delta + \nabla = L\cdot H\cdot S$

Hence proved ✓

Solved by
Labeeb Ahmad

PA

Q9

Sol

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$C_3 + C_4$

$+ C_5 (n+$

PAPER A/2015

Q9

$(E^2 - 5E + 6)y_n = n^2 - 1$

Sol $E^2 y_n - 5E \cdot y_n + 6y_n = n^2 - 1$

$y_{n+2} - 5y_{n+1} + 6y_n = n^2 - 1$

Let $y_n = Ab^n$

So the Difference Equation becomes

$Ab^{n+2} - 5Ab^{n+1} + 6Ab^n = 0$

$Ab^n[b^2 - 5b + 6] = 0$

Since $Ab^n \neq 0$ Hence $b^2 - 5b + 6 = 0$

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$b(b-2) - 3(b-2) = 0$

$(b-2)(b-3) = 0$

$b - 2 = 0$ or $b - 3 = 0$

$b = 2$ or $b = 3$

Solved by
Labeeb Ahmad

So the Complementary Solution is

$y_c = c_1(2)^n + c_2(3)^n$

Now we find Particular integral

Put $y_n = c_3 + c_4 n + c_5 n^2$

So the Difference Equation becomes

$[c_3 + c_4(n+2) + c_5(n+2)^2] - 5[c_3 + c_4(n+1) + c_5(n+1)^2] + 6[c_3 + c_4 n + c_5 n^2] = n^2 - 1$

$= f_{n+1}\Delta g_n + \cancel{f_{n+1} g_n} - \cancel{f_n g_n} + g_n \Delta f_n$

$= f_{n+1}\Delta g_n + g_n \Delta f_n$

$= R.H.S$ ✓ Hence Proved

(iii)

$\nabla - \Delta = \nabla\Delta$

Take R.H.S $\nabla\Delta f(x) = \nabla[\Delta f(x)]$

$= \nabla[f(x+h) - f(x)]$

$= \nabla f(x+h) - \nabla f(x)$

$= [f(x+h) - f(x)] - \nabla f(x)$

$= \Delta f(x) - \nabla f(x)$

$\nabla\Delta f(x) = (\Delta - \nabla) f(x)$

$\nabla\Delta = \Delta - \nabla = L.H.S$

Hence Proove

(iii)

$\Delta + \nabla = (\Delta/\nabla) - (\nabla/\Delta)$

Take R.H.S $= \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$= \frac{\Delta^2 - \nabla^2}{\nabla\Delta} = \frac{(\Delta+\nabla)(\Delta-\nabla)}{\nabla\Delta}$

$= \frac{(\Delta+\nabla)\cancel{(\Delta-\nabla)}}{\cancel{(\Delta-\nabla)}}$

$= \Delta + \nabla = L.H.S$

Hence Proved

Solved by
Labeeb Ahmad

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Q9

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2015 Q[9]

(ii)

Solved by
Labeeb Ahmad

$$y_{n+4} - A^n y_n = 2^n$$

Solution:

For (C.F)

$$y_{n+4} - A^n y_n = 0 \quad \text{---(i)}$$

Put $y_n = Ab^n$

Here for Equation (i) becomes

$$Ab^{n+4} - A^n(Ab^n) = 0$$

$$(Ab^n)\left[b^4 - A^n\right] = 0$$

$C_3 + C_4(n+2) + C_5(n^2+4n+4) - 5C_3 - 5C_4(n+1)$

$- 5C_5(n^2+1+2n) + 6C_3 + 6C_4 n + 6C_5 n^2 = n^2 - 1$

Now compare the co-efficients of n^2, n and constant.

$C_5 + 6C_5 - 5C_5 = 1$

$2C_5 = 1 \Rightarrow \boxed{C_5 = 1/2}$

Also $C_4 + 4C_5 - 5C_4 - 10C_5 + 6C_4 = 0$

$2C_4 - 6C_5 = 0$

$2C_4 - 6(\frac{1}{2}) = 0$

$2C_4 - 3 = 0$

$\boxed{C_4 = 3/2}$

Also $C_3 + 2C_4 + 4C_5 - 5C_3 - 5C_4 - 5C_5 + 6C_3 = -1$

$2C_3 - 3C_4 - C_5 = -1$

$2C_3 - 3(3/2) - (1/2) = -1$

$2C_3 - \frac{9}{2} - \frac{1}{2} = -1$

$2C_3 - 5 = -1$

$2C_3 = -1 + 5$

$2C_3 = 4$

$\boxed{C_3 = 2}$

Solved by
Labeeb Ahmad

Hence the Final solution is

$y_n = y_c + y_p$

$$\boxed{y_n = C_1 2^n + C_2 3^n + 2 + \frac{3}{2}n + \frac{n^2}{2}}$$

5/2015

Q7(i) Solve $y_{t+2} - 9y_{t+1} + 20y_t = 3^t(t^2+1)$

Solution:- For C-F

$$y_{t+2} - 9y_{t+1} + 20y_t = 0$$

Let $y_t = Ab^t$

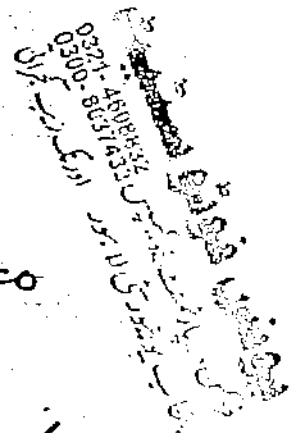
$$Ab^{t+2} - 9Ab^{t+1} + 20Ab^t = 0$$

$$Ab^t[b^2 - 9b + 20] = 0$$

Since $Ab^t \neq 0$

So $b^2 - 9b + 20 = 0$

Solved by
Labeeb Ahmad



$$b^2 - 4b - 5b + 20 = 0$$

$$b(b-4) - 5(b-4) = 0$$

$$(b-4)(b-5) = 0$$

$b - 4 = 0$ or $b - 5 = 0$

$b = 4, 5$

So the roots are real and unequal

Hence the Complemetry solution is

$$y_c = C_1(4)^t + C_2(5)^t$$

Now we find Particular Integral

Put

$$y_t = 3^t(C_3 + C_4 t + C_5 t^2)$$

Then for

$$y_{t+1} = 3^{t+1}(C_3 + C_4(t+1) + C_5(t+1)^2)$$

and $y_{t+2} = 3^{t+2}(C_3 + C_4(t+2) + C_5(t+2)^2)$

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$9C_4 + 36C_5 - 27C_4 - 54C_5 + 20C_4 = 0$

$2C_4 - 18C_5 = 0$

$C_4 - 9C_5 = 0$

$C_4 = 9C_5$

$C_4 = 9 \times \frac{1}{2}$

$\boxed{C_4 = 9/2}$

Now Comparing Constant

$9C_3 + 18C_4 + 36C_5 - 27C_3 - 27C_4 - 27C_5 + 20C_3 = 1$

$2C_3 - 9C_4 + 9C_5 = 1$

$2C_3 - 9\left(\frac{9}{2}\right) + 9\left(\frac{1}{2}\right) = 1$

$2C_3 - \frac{81}{2} + \frac{9}{2} = 1$

$2C_3 = 1 + \frac{81}{2} - \frac{9}{2}$

$2C_3 = \frac{2 + 81 - 9}{2} = \frac{74}{2} = 37$

$\boxed{C_3 = 37/2}$

Solved by Labeeb Ahmad

Putting these in question

$$3^{t+2}\left[C_3+C_4(t+2)+C_5(t+2)^2\right]-9\cdot 3^{t+1}\left[C_3+C_4(t+1)+C_5(t+1)^2\right]+20\left[3^t\left[C_3+C_4t+C_5t^2\right]\right]=3^t(t^2+1)$$

$$3^t\left\{9\left[C_3+C_4t+2C_4+C_5t^2+4C_5+4tC_5\right]-27\left[C_3+tC_4+C_4+t^2C_5+C_5+2tC_5\right]+20\left[C_3+C_4t+C_5t^2\right]\right\}=3^t(t^2+1)$$

Comparing the co-efficient "t^2"

$$9C_5-27C_5+20C_5=1$$

$$2C_5=1$$

$$C_5=1/2$$

Solved by
Labeeb Ahmad

Now Comparing the co-efficient of "t"

$$y = (C_1 + C_2 n)(2)^n$$

Now we find P.I

Put $y_n = C \cdot n^2 \cdot 2^n$

Putting this in equation (1)

$$C(n+2)^2 \cdot 2^{n+2} - 4C(n+1)^2 \cdot 2^{n+1} + 4Cn^2 \cdot 2^n = 2 \cdot 2^n$$

$$2^n \left[4C(n+2)^2 - 8C(n+1)^2 + 4Cn^2 \right] = 2 \cdot 2^n$$

$$4C(n^2+4n+4) - 8C(n^2+2n+1) + 4Cn^2 = 2$$

$$4Cn^2 + 16Cn + 16C - 8Cn^2 - 16Cn - 8C + 4Cn^2 = 2$$

$$8C = 2$$

$$\boxed{C = 1/4}$$

Solved by
Labeeb Ahmad

So $y_n = \frac{1}{4} n^2 \cdot 2^n$

Hence the general solution is

$$y_n = C.F + P.I$$

$$y_n = (C_1 + C_2 n)(2)^n + \frac{n^2}{4} \cdot (2)^n$$

So the particular solution is

$$y_t = 3^t\left[\frac{t^2}{2} + \frac{9t}{2} + \frac{37}{2}\right]$$

Hence the Final solution is

$$y = C.F + P.I$$

$$y = c_1(4)^t + c_2(5)^t + 3^t\left[\frac{t^2}{2} + \frac{9t}{2} + \frac{37}{2}\right]$$

S|2015 Q7(b)(ii)

$$y_{n+2} - 4y_{n+1} + 4y_n = 2 \cdot 2^n$$

Solution:- For C-F

$$y_{n+2} - 4y_{n+1} + 4y_n = 0$$

Put $y_n = Ab^n$

So $Ab^{n+2} - 4Ab^{n+1} + 4Ab^n = 0$

$Ab^n(b^2 - 4b + 4) = 0$

Since $Ab^n \neq 0$ So $b^2 - 4b + 4 = 0$

$b^2 - 2b - 2b + 4 = 0$

$b(b-2) - 2(b-2) = 0$

$(b-2)(b-2) = 0$

$b = 2, 2$

Now

P.

Putti

C (n+

$y^n[4($

4C(

$4Cn^2$

So -

Hence

y_n :

$y_n =$

Solved by
Labeeb Ahmad

Now we find P-I

Put $y_R = cR$ in (i)

3 — (i) $(R+3)C - 3(R+2)C + 2RC = 3$

$RC + 3C - 3RC - 6C + 2RC = 3$

$-3C = 3$

$\boxed{C = -1}$

So $y_k = -R$

Hence the Final solution is

$y_R = C.F + P.I$

$= C_1 + C_2[1+\sqrt{3}]^R + [1-\sqrt{3}]^R - k$

A|2014

Q7(ii) $y_{R+2} - 9y_{R+1} - 52y_R = 2^R + (R^2+1)$ — (i)

Solution:-

For C.F

$y_{R+2} - 9y_{R+1} - 52y_R = 0$

Let $y_R = Ab^R$

$Ab^{R+2} - 9Ab^{R+1} - 52Ab^R = 0$

$Ab^R[b^2 - 9b - 52] = 0$

Since $Ab^R \neq 0$ So $b^2 - 9b - 52 = 0$

Solved by
Labeeb Ahmad

PAPER A|2014

Q7(i) $y_{k+3} - 3y_{k+2} + 2y_k = 3$ — (i)

Sol — For C-F

$$y_{k+3} - 3y_{k+2} + 2y_k = 0$$

Let $y_k = Ab^k$

So $Ab^{k+3} - 3Ab^{k+2} + 2Ab^k = 0$

$$Ab^k[b^3 - 3b^2 + 2] = 0$$

Since $Ab^k \neq 0$

So $b^3 - 3b^2 + 2 = 0$

	1	-3	0	2
1	0	1	-2	-2
	1	-2	-2	0

$b = 1$ and $b^2 - 2b - 2 = 0$

$$b = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)}$$

$$b = \frac{2 \pm \sqrt{12}}{2}$$

$$b = \frac{2 \pm 2\sqrt{3}}{2}$$

$$b = 1 \pm \sqrt{3}$$

Hence Roots are $b = 1, 1+\sqrt{3}, 1-\sqrt{3}$

So $y_c = c_1(1)^k + c_2[1+\sqrt{3}]^k + c_3[1-\sqrt{3}]^k$

Solved by
Labeeb Ahmad

Now we find P-I

Put $y_k = c\,k$ in (i)

$(k+3)c - 3(k+2)c + 2kc = 3$

$kc + 3c - 3kc - 6c + 2kc = 3$

$-3c = 3$

$\boxed{c = -1}$

So $y_k = -k$

Hence the Final solution is

$y_k = C.F + P.I$

$= c_1 + c_2[1+\sqrt{3}]^k + [1-\sqrt{3}]^k - k$

A|2014

Q7(ii) $y_{k+2} - 9y_{k+1} - 52y_k = 2^k + (k^2+1)$ — (i)

Solution:-

For C.F

$y_{k+2} - 9y_{k+1} - 52y_k = 0$

Let $y_k = Ab^k$

$Ab^{k+2} - 9Ab^{k+1} - 52Ab^k = 0$

$Ab^k[b^2 - 9b - 52] = 0$

Since $Ab^k \neq 0$ So $b^2 - 9b - 52 = 0$

Solved by
Labeeb Ahmad

$b^2 - 13b + 4b - 52 = 0$

$b(b-13) + 4(b-13) = 0$

$(b-13)(b+4) = 0$

$b = 13, -4$

Roots are real & unequal

$y_c = C_1(13)^k + C_2(-4)^k$

Now we find P-I

P-I for 2^k

Put $y_k = C \cdot 2^k$

So

$C \cdot 2^{k+2} - 9C \cdot 2^{k+1} - 52C \cdot 2^k = 2^k$

$2^k[4C - 18C - 52] = 2^k$

$-66C = 1$

$C = -1/66$

So $y_k = -\frac{1}{66} \cdot 2^k$

Solved by
Labeeb Ahmad

Now P-I for k^2+1

Put

$y_k = C_3 + C_4 k + C_5 k^2$ — (2)

Equation becomes

$[C_3 + C_4$

$+$

$[C_3 + C_4$

$+ C_4 k +$

Compar

C_5

Compa

$C_4 + 4$

$- 6C$

$6C$

$6C$

$$[C_3 + C_4(k+2) + C_5(k+2)^2] - 9[C_3 + C_4(k+1) + C_5(k+1)^2] - 52[C_3 + C_4 k + C_5 k^2] = k^2 + 1$$

$$[C_3 + C_4 k + 2C_4 + C_5 k^2 + C_5(4k) + C_5(4)] - 9[C_3 + C_4 k + C_4 + C_5 k^2 + C_5(2k) + C_5] - 52[C_3 + C_4 k + C_5 k^2] = k^2 + 1$$

Comparing the co-efficient of "k^2"

$$C_5 - 9C_5 - 52C_5 = 1$$

$$-60C_5 = 1$$

$$\boxed{C_5 = -1/60}$$

Comparing the co-efficient of "k"

$$C_4 + 4C_5 - 9C_4 - 18C_5 - 52C_4 = 0$$

$$-60C_4 - 14C_5 = 0$$

$$60C_4 = -14C_5$$

$$60C_4 = -14\left(\frac{-1}{60}\right) = \frac{14}{60} = \frac{7}{30}$$

$$\boxed{C_4 = \frac{7}{1800}}$$

Solved by Labeeb Ahmad

A/2013

Q8 {b

Solve

Solut

Put

Since $Ab^n \neq 0$

Comparing Constants

$$C_3 + 2C_4 + 4C_5 - 9C_3 - 9C_4 - 9C_5 - 52C_3 = 1$$

$$-60C_3 - 7C_4 - 5C_5 = 1$$

$$-60C_3 - 7\left(\frac{7}{1800}\right) - 5\left(-\frac{1}{60}\right) = 1$$

$$-60C_3 - \frac{49}{1800} + \frac{1}{6} = 1$$

$$-60C_3 = 1 - \frac{1}{6} + \frac{49}{1800}$$

$$-60C_3 = \frac{1549}{1800}$$

Solved by
Labeeb Ahmad

$$C_3 = -\frac{1}{60} \times \frac{1549}{1800}$$

$$C_3 = \frac{-1549}{108000}$$

Putting the values of C_3, C_4 and C_5 in (2)

$$y_k = -\frac{k^2}{60} + \frac{7k}{1800} - \frac{1549}{108000}$$

Hence the Final Solution is

$$y = C.F + P.I$$

$$y = C_1(13)^k + C_2(-4)^k - \frac{2^k}{66} - \frac{k^2}{60} + \frac{7k}{1800} - \frac{1549}{108000}$$

Solved by
Labeeb Ahmad

$R =$

$= \sqrt{}$

$\theta = \tan$

$= \tan$

2013

Q8 {b}

Solve $y_{n+2} - y_{n+1} + 6y_n = 0$

$y_1 = 5$, $y_2 = 9$

Solution :-

Given $y_{n+2} - y_{n+1} + 6y_n = 0$

Put $y_n = Ab^n$

$Ab^{n+2} - Ab^{n+1} + 6Ab^n = 0$

$Ab^n[b^2 - b + 6] = 0$

$\because Ab^n \neq 0$, $\therefore b^2 - b + 6 = 0$

$$b = \frac{1 \pm \sqrt{1 - 4(1)(6)}}{2}$$

$$b = \frac{1 \pm \sqrt{-23}}{2}$$

$$b = \frac{1 \pm \sqrt{23}\,i}{2}$$

$$b = \frac{1}{2} + i\frac{\sqrt{23}}{2}, \quad \frac{1}{2} - i\frac{\sqrt{23}}{2}$$

$$R = \sqrt{x^2 + y^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{23}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{23}{4}}$$

$$= \sqrt{\frac{24}{4}} = \sqrt{6}$$

$$\theta = \tan^{-1} y/x$$

$$= \tan^{-1}\left(\frac{\sqrt{23}}{2} / \frac{1}{2}\right) = \tan^{-1}\sqrt{23}$$

Solved by Labeeb Ahmad

And $\theta = \tan^{-1}\left(\frac{-\sqrt{23}}{\frac{1}{2}}\right)$

$= \tan^{-1}(-\sqrt{23})$

$y_n = (\sqrt{6})^n\left[A\cos(\tan^{-1}\sqrt{23})n + B\sin(\tan^{-1}\sqrt{23})n\right]$

$+(\sqrt{6})^n\left[C\cdot\cos n(\tan^{-1}\sqrt{23}) + D\sin n(\tan^{-1}\sqrt{23})\right]$

A/20
Q9(a

Solution

Put y_t

So

Since

Solv
Labeeb

Roots

So $y_c =$

Now

Put

Put in (

$4^{t+2}[(t+$

$(t+1)C_3 +$

2013

9(a) Solve $y_{t+2} - 9y_{t+1} + 20y_t = 4^t(t^2+1)$ - ①

Solution : For C-F

$$y_{t+2} - 9y_{t+1} + 20y_t = 0$$

Put $y_t = Ab^t$

So $Ab^{t+2} - 9Ab^{t+1} + 20Ab^t = 0$

$$Ab^t[b^2 - 9b + 20] = 0$$

Since $Ab^t \neq 0$ So $b^2 - 9b + 20 = 0$

$$b^2 - 4b - 5b + 20 = 0$$

$$b(b-4) - 5(b-4) = 0$$

$$(b-4)(b-5) = 0$$

$$b = 4, 5$$

Solved by Labeeb Ahmad

Roots are real and unequal

So $y_c = C_1(4)^t + C_2(5)^t$

Now we find P-I

Put $y_t = 4^t[tC_3 + t^2C_4 + t^3C_5]$

Put in ①

$$4^{t+2}[(t+2)C_3 + (t+2)^2C_4 + (t+2)^3C_5] - 9\cdot 4^{t+1}[(t+1)C_3 + (t+1)^2C_4 + (t+1)^3C_5] + 20\cdot 4^t[tC_3 + t^2C_4 + t^3C_5] = 4^t(t^2+1)$$

$$4^t\{16[(t+2)C_3+(t^2+4t+4)C_4+(t^3+6t^2+12t+8)C_5]$$
$$-36[(t+1)C_3+(t^2+2t+1)C_4+(t^3+3t^2+3t+1)C_5]$$
$$+20[tC_3+t^2C_4+t^3C_5]\} = 4^t(t^2+1)$$

Comparing the Co-efficient of "t^2"

$$16C_4+96C_5-36C_4-108C_5+20C_4=1$$

$$-12C_5=1$$

$$\boxed{C_5=-1/12}$$

Comparing the Co-efficient of "t"

$$16C_3+64C_4+192C_5-36C_3-72C_4$$
$$-108C_5+20C_3=0$$

$$8C_4+84C_5=0$$

$$-8C_4+84\left(\frac{-1}{12}\right)=0$$

$$-8C_4-7=0$$

$$\boxed{C_4=-7/8}$$

Now
$32C_3$
$-4C$
$-4C_3$
$-4C$
$-4C$
$-4C$
$\boxed{C_3}$

So P=
$y_t=$
Hence
$y_t=($
$y_t=C_1$

Solved by:
Labeeb Ahmad

Now Comparing Constant

$$32C_3 + 64C_4 + 128C_5 - 36C_3 - 36C_4 - 36C_5 = 1$$

$$-4C_3 + 28C_4 + 92C_5 = 1$$

$$-4C_3 + 28\left[-\frac{7}{8}\right] + 92\left[-\frac{1}{12}\right] = 1$$

$$-4C_3 - \frac{49}{2} - \frac{23}{3} = 1$$

$$-4C_3 = 1 + \frac{49}{2} + \frac{23}{3}$$

$$-4C_3 = \frac{199}{6}$$

$$\boxed{C_3 = -199/24}$$

Solved by
Labeeb Ahmad

So P-I is

$$y_t = 4^t\left[-\frac{t^3}{12} - \frac{7t^2}{8} - \frac{199}{24}\right]$$

Hence Final solution is

$$y_t = C.F + P.I$$

$$y_t = C_1(4)^t + C_2(5)^t + 4^t\left[-\frac{t^3}{12} - \frac{7t^2}{8} - \frac{199}{24}\right]$$

PAPER A12012

Q8(a) Solve

$$y_{n+3} - 2y_{n+1} - 4y_n = 2^n \quad \text{——} (i)$$

Solution For C-F

$$y_{n+3} - 2y_{n+1} - 4y_n = 0$$

Let $y_n = Ab^n$

$$Ab^{n+3} - 2Ab^{n+1} - 4Ab^n = 0$$

$$Ab^n[b^3 - 2b - 4] = 0$$

Since $Ab^n \neq 0$; $b^3 - 2b - 4 = 0$

	1	0	−2	−4
2	0	2	4	4
	1	2	2	0

$b = 2$ and $b^2 + 2b + b = 0$

$$b = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$$

$$b = \frac{-2 \pm \sqrt{-4}}{2}$$

$$b = \frac{-2 \pm 2i}{2}$$

$$b = -1 \pm i$$

$$b = -1 + i \;,\; -1 - i$$

$$R = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}(y/x) = \tan(1/-1) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

because $(-1, 1)$ belong II (Quad) $\theta = \frac{3\pi}{4}$

Solved by Labeeb Ahmad

Also

So i

$y_c = (\sqrt{2}$

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$y_c = (\sqrt{2}$

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put

put

Also $\theta = \tan^{-1}(y/x) = \tan^{-1}(-1/-1) = \tan^{-1}(1) = \frac{\pi}{4}$

because $(-1,-1)$ belong 3rd(qud) $\theta = \frac{5\pi}{4}$

(i)

So the Complementry Solution is

$$y_c = (\sqrt{2})^n \left[C_1 \cos\left(\frac{3\pi}{4}\right) + C_2 i \sin\left(\frac{3\pi}{4}\right) \right]$$

$$+ (\sqrt{2})^n \left[C_3 \cos\left(\frac{5\pi}{4}\right) + C_4 i \sin\left(\frac{5\pi}{4}\right) \right]$$

$$y_c = (\sqrt{2})^n \left[C_1\left(-\frac{1}{\sqrt{2}}\right) + C_2 i \left(\frac{1}{\sqrt{2}}\right) \right]$$

$$+ (\sqrt{2})^n \left[C_3\left(-\frac{1}{\sqrt{2}}\right) + C_4 i \left(-\frac{1}{\sqrt{2}}\right) \right]$$

$$y_c = (\sqrt{2})^n \cdot \frac{1}{\sqrt{2}} \left[-C_1 + C_2 i - C_3 - C_4 i \right]$$

$$y_c = (\sqrt{2})^{n-1} \left[(-C_1 - C_3) + i(C_2 - C_4) \right]$$

Put $-C_1 - C_3 = \alpha$

and $C_2 - C_4 = \beta$

$$y_c = (\sqrt{2})^{n-1} \cdot [\alpha + i\beta]$$

Now we find $P.I$

Put $y_n = c \cdot n \cdot 2^n$

put in (i)

Solved by Labeeb Ahmad

$C(n+3)\cdot 2^{n+3} - 2C(n+1)2^{n+1} - 4Cn\cdot 2^{n} = 2^{n}$

$2^{n}[8C(n+3) - 4C(n+1) - 4Cn] = 2^{n}$

$8Cn + 24C - 4Cn - 4C - 4Cn = 1$

$20C = 1$

$$C = \frac{1}{20}$$

Solved by
Labeeb Ahmad

So $y_n = \frac{n}{20}\cdot 2^{n}$

Hence the Final Solution is

$y_n = C\text{-}F + P\text{-}I$

$y_n = (\sqrt{2})^{n-1}(\alpha + i\beta) + \frac{n}{20}\cdot 2^{n}$

Fibonacci Difference Equ

A/2012 , A/2009 Q7(a)

Q8(b) Form the Fibonacci Difference Equation and solve it.

Solution :-

The Fibonacci number of ... interesting sequence in which each num... is equal to sum of perivious two ...

The ... are ... Let ... sequen... Put ... So ... Since ... So ... So ... $y_n = C$ ... Take ... $y_1 = 1$

The integers 0, 1, 1, 2, 3, 5, 8, 13, 21 - - - - - are said to form a Fibonacci sequence

Let y_n be the nth term of sequence $y_n = y_{n-2} + y_{n-1}$ if $n > 2$

$$y_{n+2} = y_{n+1} + y_n \text{ if } n > 0$$

$$y_{n+2} - y_{n+1} - y_n = 0$$

Put $y_n = Ab^n$

So $Ab^{n+2} - Ab^{n+1} - Ab^n = 0$

$$Ab^n [b^2 - b - 1] = 0$$

Since $Ab^n \neq 0$

So $b^2 - b - 1 = 0$

$$b = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$b = \frac{1 \pm \sqrt{5}}{2}$$

So the Complementry solution is

$$y_n = C_1 \left[\frac{1+\sqrt{5}}{2}\right]^n + C_2 \left[\frac{1-\sqrt{5}}{2}\right]^n \quad \text{---- (1)}$$

Take n = 1

$$y_1 = C_1 \left[\frac{1+\sqrt{5}}{2}\right] + C_2 \left[\frac{1-\sqrt{5}}{2}\right] = 0 \quad \text{---- (iii)}$$

Solved by Labeeb Ahmad

Take $n=2$

$$y_2 = C_1\left[\frac{1+\sqrt{5}}{2}\right]^2 + C_2\left[\frac{1-\sqrt{5}}{2}\right]^2 = 1 \quad \text{——— (iv)}$$

Equation (iii) multiply by $\left[\frac{1+\sqrt{5}}{2}\right]$

Then subtract equation (iv)

$$C_1\left[\frac{1+\sqrt{5}}{2}\right]^2 + C_2\left[\frac{1-\sqrt{5}}{2}\right]\left[\frac{1+\sqrt{5}}{2}\right] = 0$$

$$- C_1\left[\frac{1+\sqrt{5}}{2}\right]^2 + C_2\left[\frac{1-\sqrt{5}}{2}\right]^2 = 1$$

$$C_2\left[\frac{1-(\sqrt{5})^2}{4}\right] - C_2\left[\frac{1+5-2\sqrt{5}}{4}\right] = -1$$

$$C_2\left[\frac{1-5}{4}\right] - C_2\left[\frac{6-2\sqrt{5}}{4}\right] = -1$$

$$C_2\left[\frac{-4}{4}\right] - C_2\left[\frac{3-\sqrt{5}}{2}\right] = -1$$

$$-C_2 - C_2\left[\frac{3-\sqrt{5}}{2}\right] = -1$$

$$+C_2\left[1+\frac{3-\sqrt{5}}{2}\right] = +1$$

$$C_2\left[\frac{2+3-\sqrt{5}}{2}\right] = 1$$

Solved by
Labeeb Ahmad

$$C_2\left[\frac{5-\sqrt{5}}{2}\right] = 1$$

$$C_2 = \left[\frac{2}{5-\sqrt{5}}\right] \times \frac{5+\sqrt{5}}{5+\sqrt{5}}$$

$$C_2 = \frac{(2)(5+\sqrt{5})}{(5)^2-(\sqrt{5})^2}$$

$$C_2 = \frac{2(5+\sqrt{5})}{25-5}$$

$$C_2 = \frac{2(5+\sqrt{5})}{20}$$

$$\boxed{C_2 = \frac{5+\sqrt{5}}{10}}$$

Solved by Labeeb Ahmad

Put the value of C_2 in (iii)

$$C_1\left[\frac{1+\sqrt{5}}{2}\right] + \left[\frac{5+\sqrt{5}}{10}\right]\left[\frac{1-\sqrt{5}}{2}\right] = 0$$

$$C_1\left[\frac{1+\sqrt{5}}{2}\right] + \left[\frac{5-5\sqrt{5}+\sqrt{5}-5}{20}\right] = 0$$

$$C_1\left[\frac{1+\sqrt{5}}{2}\right] + \left[\frac{-4\sqrt{5}}{20}\right] = 0$$

$$C_1\left[\frac{1+\sqrt{5}}{2}\right]+\left[\frac{-\sqrt{5}}{5}\right]=0$$

$$C_1\left[\frac{1+\sqrt{5}}{2}\right]+\left[\frac{-1}{\sqrt{5}}\right]=0$$

$$C_1=\frac{1}{\sqrt{5}}\times\frac{2}{1+\sqrt{5}}$$

$$C_1=\frac{2}{\sqrt{5}+(\sqrt{5})^2}$$

$$C_1=\frac{2}{5+\sqrt{5}}\times\frac{5-\sqrt{5}}{5-\sqrt{5}}$$

$$C_1=\frac{2(5-\sqrt{5})}{(5)^2-(\sqrt{5})^2}$$

$$C_1=\frac{2(5-\sqrt{5})}{25-5}$$

$$C_1=\frac{2(5-\sqrt{5})}{20}$$

$$\boxed{C_1=\frac{5-\sqrt{5}}{10}}$$

Solved by
Labeeb Ahmad

Put the value of C_1 and C_2 in (i)

$$y_n=\left[\frac{5-\sqrt{5}}{10}\right]\left[\frac{1+\sqrt{5}}{2}\right]^n+\left[\frac{5+\sqrt{5}}{10}\right]\left[\frac{1-\sqrt{5}}{2}\right]^n$$

S/2011

Q9(a)

Solve the Difference equation

$$y_{k+2} - 5y_{k+1} + 6y_k = 2^k(k^2+1) \quad \text{---} (i)$$

Solution: For C-F

$$y_{k+2} - 5y_{k+1} + 6y_k = 0$$

let $y_k = Ab^k$

$$Ab^{k+2} - 5Ab^{k+1} + 6Ab^k = 0$$

$$Ab^k[b^2 - 5b + 6] = 0$$

Since $Ab^k \neq 0$ So $b^2 - 5b + 6 = 0$

$$b^2 - 2b - 3b + 6 = 0$$

$$b(b-2) - 3(b-2) = 0$$

$$(b-2)(b-3) = 0$$

$$b = 2, 3$$

Roots are real and unequal

Therefore

$$y_c = C_1(2)^k + C_2(3)^k$$

Now we find P-I

Put $y_k = 2^k[kC_3 + k^2C_4 + k^3C_5]$

putting in (i)

$$2^{k+2}[(k+2)C_3 + (k+2)^2C_4 + (k+2)^3C_5] - 5 \cdot 2^{k+1}$$

$$[(k+1)C_3 + (k+1)^2C_4 + (k+1)^3C_5] + 6 \cdot 2^k[kC_3 + k^2C_4 + k^3C_5] = 2^k(k^2+1)$$

Solved by Labeeb Ahmad

$$7^k\{4[(k+2)C_3+(k^2+4k+4)C_4+(k^3+6k^2+12k+8)C_5]$$
$$-10[(k+1)C_3+(k^2+2k+1)C_4+(k^3+3k^2+3k+1)C_5]$$
$$+6[kC_3+k^2C_4+k^3C_5]=7^k[k^2+1]$$

Comparing the Co-efficient of "k"

$$4C_4+24C_5-10C_4-30C_5+6C_4=1$$
$$-6C_5=1$$
$$\boxed{C_5=-1/6}$$

Now Comparing the Co-efficient of "k"

$$4C_3+16C_4+48C_5-10C_3-20C_4-30C_5$$
$$+6C_3=0$$
$$-4C_4+18C_5=0$$
$$-4C_4+18\left(-\frac{1}{6}\right)=0$$
$$-4C_4-3=0$$
$$\boxed{C_4=\frac{-3}{4}}$$

Now Compar Constant

$$8C_3+16C_4+32C_5-10C_3-10C_4-10C_5$$

$$-2C$$
$$-2C_3$$
$$-2C$$
$$-2C$$
$$-2C$$
$$\boxed{C}$$

So y_k

Hence

$y_k =$

$y_k =$

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$12k+8)C_5]$ $\quad -2C_3 + 6C_4 + 22C_5 = 1$

$3k+1)C_5]$ $\quad -2C_3 + 6\left(\frac{-3}{4}\right) + 22\left(\frac{-1}{6}\right) = 1$

$k^2+1]$ $\quad -2C_3 - \frac{9}{2} - \frac{11}{3} = 1$

$= "k^2"$ $\quad -2C_3 = 1 + \frac{9}{2} + \frac{11}{3}$

$+6C_4 = 1$ $\quad -2C_3 = \frac{55}{6}$

$$\boxed{C_3 = -55/12}$$

of "k" So $y_k = 2^k\left[-\frac{k^3}{6} - \frac{3k^2}{4} - \frac{55}{12}k\right]$

$4+30C_5$ Hence the Final solution is

$y_k = C.F + P.I$

$y_k = C_1(2)^k + C_2(3)^k$

$+ 2^k\left[-\frac{k^3}{6} - \frac{3k^2}{4} - \frac{55}{12}k\right]$

by
hmad

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$C_5 = 1$

S/2011

Q9(b) Solve the following Difference equation $y_{n+2} + 13y_{n+1} + 3y_n = 3^n(\sin 2n)$ — (i)

Solution :- For C-F

$$y_{n+2} + 13y_{n+1} + 3y_n = 0$$

Put $y_n = Ab^n$

$$Ab^{n+2} + 13Ab^{n+1} + 3Ab^n = 0$$

$$Ab^n[b^2 + 13b + 3] = 0$$

Since $Ab^n \neq 0$ So $b^2 + 13b + 3 = 0$

$$b = \frac{-13 \pm \sqrt{169 - 4(1)(3)}}{2(1)}$$

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$$b = \frac{-13 \pm \sqrt{169 - 12}}{2}$$

$$b = \frac{-13 \pm \sqrt{157}}{2}$$

Hence the Complementry solution is

$$y_c = C_1\left[\frac{-13+\sqrt{157}}{2}\right]^n + C_2\left[\frac{-13-\sqrt{157}}{2}\right]^n$$

Now we find P-I

Put $y = 3^n[C_3 \sin 2n + C_4 \cos 2n]$

Eq(i) becomes

$3^{n+2}[C_3 \sin 2(n+2) + C_4 \cos 2(n+2)] + 13 \cdot 3^{n+1}$

$[C_3 \sin 2(n+1) + C_4 \cos 2(n+1)] + 3 \cdot 3^n [C_3 \sin 2n + C_4 \cos 2n] = 3^n \sin 2$

$3^n \{9[C_3 \sin(2n+4) + C_4 \cos(2n+4)]$

$+ 39[C_3 \sin(2n+2) + C_4 \cos(2n+2)]$

$+ 3[C_3 \sin 2n + C_4 \cos 2n]\} = 3^n \sin 2n$

$9[C_3(\sin 2n \cdot \cos 4 + \cos 2n \cdot \sin 4) + C_4(\cos 2n \cdot \cos 4$

$- \sin 2n \cdot \sin 4)] + 39[C_3(\sin 2n \cdot \cos 2 + \cos 2n \cdot \sin 2)$

$+ C_4(\cos 2n \cdot \cos 2 - \sin 2n \cdot \sin 2)] + 3[C_3 \sin 2n$

$+ C_4 \cos 2n] = \sin 2n$

$9C_3 \cdot \sin 2n \cdot \cos 4 + 9C_3 \cos 2n \cdot \sin 4 + 9C_4 \cos 2n \cdot \cos$

$- 9C_4 \sin 2n \cdot \sin 4 + 39C_3 \sin 2n \cdot \cos 2 +$

$39C_3 \cos 2n \cdot \sin 2 + 39C_4 \cos 2n \cdot \cos 2 - 39C_4$

$\sin 2n \cdot \sin 2 + 3C_3 \sin 2n + 3C_4 \cos 2n = \sin 2n$

$(9C_3 \cdot \cos 4 - 9C_4 \cdot \sin 4 + 39C_3 \cdot \cos 2 - 39C_4 \cdot \sin 2$

$+ 3C_3) \sin 2n$

$(9C_3 \sin 4 + 9C_4 \cdot \cos 4 + 39C_3 \cdot \sin 2 + 39C_4 \cdot \cos 2$

$+ 3C_4) \cos 2n = \sin 2n$

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Comparing the co-efficients of sin2n and cos2n

$9C_3\cos 4 - 9C_4 \cdot \sin 4 + 3C_3 \cdot \cos 2 - 3C_4 \cdot \sin 2$

$+3C_3 = 1 \quad \text{---} (1)$

$9C_3 \sin 4 + 9C_4 \cos 4 + 3C_3 \sin 2 + 3C_4 \cos 2$

$+3C_4 = 0 \quad \text{---} (2)$

$(9\cos 4 + 3\cos 2 + 3)C_3 - (9\sin 4 + 3\sin 2)C_4 = 1 \quad (3)$

$(9\sin 4 + 3\sin 2)C_3 + (9\cos 4 + 3\cos 2 + 3)C_4 = 0$

Let $9\cos 4 + 3\cos 2 + 3 = l_1$

And $9\sin 4 + 3\sin 2 = l_2$

Equation (3) & (4) takes a form

$l_1 C_3 - l_2 C_4 = 1$

$l_2 C_3 + l_1 C_4 = 0$

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Labeeb Ahmad

OR

$l_1 C_3 - l_2 C_4 - 1 = 0$

$l_2 C_3 + l_1 C_4 + 0 = 0$

$$\frac{C_3}{0 + l_1} = \frac{C_4}{-l_2 - 0} = \frac{1}{l_1^2 + l_2^2}$$

So $\frac{C_3}{l_1}$

C_3

So th

$y_n = \int \frac{1}{l_1^2 +$

Hence

$y_n = C_1 \left[\right.$

So $\frac{C_3}{l_1} = \frac{1}{l_1^2+l_2^2}$ and $\frac{C_4}{l_2} = \frac{1}{l_1^2+l_2^2}$

$C_3 = \frac{l_1}{l_1^2+l_2^2}$ and $C_4 = \frac{-l_2}{l_1^2+l_2^2}$

So the particular solution is

$$y_n = \left[\frac{l_1}{l_1^2+l_2^2}\sin 2n + \frac{-l_2}{l_1^2+l_2^2}\cos 2n\right]3^n$$

Hence the general solution is

$$y_n = C_1\left[\frac{-13+\sqrt{157}}{2}\right]^n + C_2\left[\frac{-13-\sqrt{157}}{2}\right]^n + 3^n\left[\frac{l_1}{l_1^2+l_2^2}\sin 2n - \frac{l_2}{l_1^2+l_2^2}\cos 2n\right]$$

where $l_1 = 9\cos 4 + 39\cos 2 + 3$

and $l_2 = 9\sin 4 + 39\sin 2$

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A/2011

Q9(a) Solve the Difference equation

$$y_{n+4} + 4y_n = 2^n \quad \text{---} (1)$$

Solution:- For C.F

$$y_{n+4} + 4y_n = 0$$

Let $y_n = Ab^n$

So $Ab^{n+4} + 4Ab^n = 0$

$$Ab^n [b^4 + 4] = 0$$

Since $Ab^n \neq 0$ so $b^4 + 4 = 0$

$$(b^2)^2 + (2)^2 + 4b^2 - 4b^2 = 0$$

Solved by Labeeb Ahmad

$$[(b^2)^2 + (2)^2 + 2(b^2)(2)] - 4b^2 = 0$$

$$(b^2+2)^2 - (2b)^2 = 0$$

$$(b^2 + 2 - 2b)(b^2 + 2 + 2b) = 0$$

$$b^2 + 2 - 2b = 0 \quad , \quad b^2 + 2b + 2 = 0$$

using Quadratic formula

$b^2 - 2b + 2 = 0$

$$b = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$$

$$b = \frac{2 \pm \sqrt{4-8}}{2}$$

$$b = \frac{2 \pm 2i}{2}$$

$$b = 1 \pm i$$

$b^2 + 2b + 2 = 0$

$$b = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$$

$$b = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$b = \frac{-2 \pm 2i}{2}$$

$$b = -1 \pm i$$

For $b = 1 \pm i$

$$R = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(1/-1) = \tan^{-1}(-1)$$

$$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

And

$$\theta = \tan^{-1}(1/1) = \tan^{-1}(1) = \frac{\pi}{4}$$

since both roots are imaginary, so the Complementry solution is

$$y_n = (\sqrt{2})^n \left[C_1 \cos n \cdot \frac{\pi}{4} + i C_2 \sin n \frac{\pi}{4}\right]$$

$$(\sqrt{2})^n \left[C_3 \cdot \cos n \cdot \frac{3\pi}{4} + i C_4 \sin n \cdot \frac{3\pi}{4}\right]$$

Now we find P.I

Put $y_n = C_5 + nC_6$

Equation (1) becomes

$$\left[C_5 + (n+4)C_6\right] + 4\left[C_5 + nC_6\right] = 2n$$

$$C_5 + nC_6 + 4C_6 + 4C_5 + 4nC_6 = 2n$$

$$5nC_6 + 5C_5 + 4C_6 = 2n$$

Comparing the Co-efficient of "n"

$$5C_6 = 2 \Rightarrow \boxed{C_6 = 2/5}$$

Solved by
Labeeb Ahmad

Now Compare Constant

$5C_5 + 4C_6 = 0$

$5C_5 + 4\left(\frac{2}{5}\right) = 0$

$5C_5 + \frac{8}{5} = 0$

$5C_5 = -\frac{8}{5}$

$\boxed{C_5 = -8/25}$

Solved by
Labeeb Ahmad

So the P-I is

$$y_n = \frac{2n}{5} - \frac{8}{25}$$

Hence the Final solution is

$y_n = C\text{-}F + P\text{-}I$

$$y_n = (\sqrt{2})^n\left[C_1 \cdot \cos \cdot n \cdot \frac{\pi}{4} + i\, C_2 \cdot \sin n \cdot \frac{\pi}{4}\right]$$

$$(\sqrt{2})^n\left[C_3 \cos n \cdot \frac{3\pi}{4} + i\, C_4 \sin n \cdot \frac{3\pi}{4}\right]$$

$$+ \frac{2n}{5} - \frac{8}{25}$$

A/2011 Q9(b)

Solve the following system of Linear Difference Equation:

$$3y_{n+1} - 2y_n + z_n = n^2 \quad \text{——} (1)$$

$$4z_{n+1} - 3z_n + y_n = 3^n + n \quad \text{——} (2)$$

Solution:- Using shift Operator, we get

$$(3E-2)y_n + z_n = n^2 \quad \text{——} (3)$$

$$(4E-3)z_n + y_n = 3^n + n \quad \text{——} (4)$$

multiplying Equation (4) by $(3E-2)$ and then subtracting from (3)

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$$(3E-2)y_n + z_n = n^2$$

$$\pm(3E-2)y_n \pm (3E-2)(4E-3)z_n = (3E-2)(3^n+n)$$

$$z_n - (3E-2)(4E-3)z_n = n^2 - (3E-2)(3^n+n)$$

$$z_n - (12E^2 - 17E + 6)z_n = n^2 - [3 \cdot E(3^n) - 2 \cdot 3^n + 3E(n) - 2n]$$

$$z_n - 12z_{n+2} + 17z_{n+1} - 6z_n = n^2 - [3(1+\Delta)3^n - 2 \cdot 3^n + 3(1+\Delta)n - 2n]$$

$$-12z_{n+2} + 17z_{n+1} - 5z_n = n^2 - [3(3^n + \Delta 3^n) - 2 \cdot 3^n + 3(n + \Delta n) - 2n]$$

$-(12Z_{n+2} - 17Z_{n+1} + 5Z_n) = n^2 - [3(3^n + 3^{n+1} - 3^n)$

$- 2\cdot 3^n + 3(n + n+1 - n) - 2n]$

$-(12Z_{n+2} - 17Z_{n+1} + 5Z_n) = n^2 - [3\cdot 3^{n+1} - 2\cdot 3^n$

$+ 3n + 3 - 2n]$

$= n^2 - [3^{n+2} - 2\cdot 3^n + 3 + n]$

$= n^2 - [9\cdot 3^n - 2\cdot 3^n + 3 + n]$

$= -(n^2 + [7\cdot 3^n + 3 + n]$

multiply both side by "-1"

$12Z_{n+2} - 17Z_{n+1} + 5Z_n = 7\cdot 3^n + (3 + n - n^2)$ — (1)

For C-F

put $Z_n = Ab^n$

$12Ab^{n+2} - 17Ab^{n+1} + 5Ab^n = 0$

$Ab^n [12b^2 - 17b + 5] = 0$

Since $Ab^n \neq 0$

So $12\cdot b^2 - 17b + 5 = 0$

$12b^2 - 12b - 5b + 5 = 0$

$12b(b-1) - 5(b-1) = 0$

$(b-1)(12b-5) = 0$

$b = 1, 5/12$

Hence the C-F is

$Z_n = C_1(1)^n + C_2(5/12)^n$

Now

P-I

Equa

$12C_3$

$3^n[12.$

$(1$

Hence

Now

Re

Equati

$12[(n+1$

$-17[(n+$

$5[n$

$[(n+$

$[(n+1)$

$[nC_4$

Solved by
Labeeb Ahmad

Now we find P-I

P-I for $7 \cdot 3^n$

Put $Z_n = C_3 3^n$

Equation (5) takes a form

$$12C_3 \cdot 3^{n+2} - 17C_3 \cdot 3^{n+1} + 5C_3 \cdot 3^n = 7 \cdot 3^n$$

$$3^n [12 \cdot C_3 (9) - 17C_3 (3) + 5C_3] = 7 \cdot 3^n$$

$$(108 - 51 + 5) C_3 = 7$$

$$62 C_3 = 7$$

$$C_3 = \frac{7}{62}$$

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Hence $Z_n = \frac{7}{62} \cdot 3^n$

Now we find P-I for $-n^2 + n + 3$

Put $Z_n = nC_4 + n^2 C_5 + n^3 C_6$ — (6)

Equation (5) becomes

$$12[(n+2)C_4 + (n+2)^2 C_5 + (n+2)^3 C_6]$$

$$-17[(n+1)C_4 + (n+1)^2 C_5 + (n+1)^3 C_6]$$

$$+5[nC_4 + n^2 C_5 + n^3 C_6] = -n^2 + n + 3$$

$$12[(n+2)C_4 + (n^2+4n+4)C_5 + (n^3+6n^2+12n+8)C_6]$$

$$-17[(n+1)C_4 + (n^2+2n+1)C_5 + (n^3+3n^2+3n+1)C_6]$$

$$+5[nC_4 + n^2 C_5 + n^3 C_6] = -n^2 + n + 3$$

Comparing the Co-efficient of "n"

$12C_5 + 72C_6 - 17C_5 - 51C_6 + 5C_5 = -1$

$21C_6 = -1$

$$\boxed{C_6 = \frac{-1}{21}}$$

Now Compare the Co-efficient of "n"

$12C_4 + 48C_5 + 144C_6 - 17C_4 - 34C_5 - 51C_6 + 5C_4 = 1$

$14C_5 + 93C_6 = 1$

$14C_5 + 93\left(\frac{-1}{21}\right) = 1$ Putting

$14C_5 + \left(\frac{-93}{21}\right) = 1$

$14C_5 = 1 + \frac{93}{21}$

$14C_5 = \frac{38}{7}$

$C_5 = \frac{38}{7 \times 14}$

$$\boxed{C_5 = \frac{19}{49}}$$

Hence

$Z_n =$

$Z_n =$

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Now Compare the Constant

$24C_4 + 48C_5 + 96C_6 - 17C_4 - 17C_5 - 17C$

$7C_4 + 31C_5 + 79C_6 = 3$

$$7C_4 + 31\left(\frac{19}{49}\right) + 79\left(-\frac{1}{21}\right) = 3$$

$$7C_4 + \frac{589}{49} - \frac{79}{21} = 3$$

$$7C_4 = 3 - \frac{589}{49} + \frac{79}{21}$$

$$7C_4 = -\frac{773}{147}$$

$$C_4 = \frac{-773}{147 \times 7} \Rightarrow \boxed{C_4 = \frac{-773}{1029}}$$

Putting the values C_4, C_5, C_6 in (6)

$$z_n = -\frac{n^3}{21} + \frac{19}{49}n^2 - \frac{773}{1029}n$$

Hence the Final Solution is

$$z_n = C.F + P.I$$

$$z_n = C_1(1)^n + C_2\left(\frac{5}{12}\right)^n + \frac{7}{62}\cdot 3^n + \left[-\frac{n^3}{21} + \frac{19}{49}n^2 - \frac{773}{1029}n\right]$$

Solved by
Labeeb Ahmad

A/2010

Q7(a) Solve the Difference Equation

$$y_{k+2} - 4y_{k+1} + 4y_k = 2^k(k^2+1) \quad \text{—(1)}$$

Solution :- For C-F

$$y_{k+2} - 4y_{k+1} + 4y_k = 0$$

put $y_k = Ab^k$

So $Ab^{k+2} - 4Ab^{k+1} + 4Ab^k = 0$

$$Ab^k[b^2 - 4b + 4] = 0$$

Since $Ab^k \neq 0$ So $b^2 - 4b + 4 = 0$

$$b^2 - 2(b)(2) + 2^2 = 0$$

$$(b-2)^2 = 0$$

$$(b-2)(b-2) = 0$$

$$b = 2, 2$$

Hence Roots are real and equal

So $y_c = (C_1 + C_2 k)(2)^k$ ...(2)

Now we find P-I

put $y_k = 2^k(k^2C_3 + k^3C_4 + k^4C_5)$

Equation (1) becomes

$$2^{k+2}[(k+2)^2C_3 + (k+2)^3C_4 + (k+2)^4C_5] - 4\cdot 2^{k+1}[(k+1)^2C_3 + (k+1)^3C_4 + (k+1)^4C_5] + 4\cdot 2^k[k^2C_3 + k^3C_4 + k^4C_5] = 2^k[k^2+1]$$

Solved by Labeeb Ahmad

Equation (1) —

$$2^k\{4[(k^2+4k+4)C_3+(k^3+6k^2+12k+8)C_4+(k^4+8k^3+24k^2+32k+16)C_5]-8[(k^2+2k+1)C_3+(k^3+3k^2+3k+1)C_4+(k^4+4k^3+6k^2+4k+1)C_5]+4[k^2C_3+k^3C_4+k^4C_5]\}=2^k(k^2+1)$$

Now Comparing the Co-efficient of "k^2"

$$4C_3+24C_4+96C_5-8C_3-24C_4-48C_5$$

$$4C_3=1$$

$$96C_5-48C_5=1$$

$$48C_5=1$$

$$\boxed{C_5=1/48}$$

Solved by Labeeb Ahmad

Now Compare the Co-efficient of "k"

$$16C_3+48C_4+48C_5-16C_3-24C_4-32C_5=0$$

$$24C_4+96C_5=0$$

$$24C_4+96\left(\frac{1}{48}\right)=0$$

$$24C_4+2=0$$

$$C_4=-\frac{2}{24}\Rightarrow\boxed{C_4=\frac{-1}{12}}$$

Now Compare Constant

$$16C_3 + 32C_4 + 64C_5 - 8C_3 - 8C_4 - 8C_5 = 1$$

$$8C_3 + 24C_4 + 56C_5 = 1$$

$$8C_3 + 24\left(-\frac{1}{12}\right) + 56\left(\frac{1}{48}\right) = 1$$

$$8C_3 - 2 + \frac{7}{6} = 1$$

$$8C_3 = 1 + 2 - \frac{7}{6} = \frac{11}{6}$$

$$\boxed{C_3 = \frac{11}{48}}$$

Solved by Labeeb Ahmad

Hence P-I is

$$y_k = 2^k\left[\frac{k^4}{48} - \frac{k^3}{12} + \frac{11k^2}{48}\right]$$

Hence General Solution is

$$y_k = C\text{-}F + P\text{-}I$$

$$y_k = (C_1 + C_2 k)(2)^k + 2^k\left[\frac{k^4}{48} - \frac{k^3}{12} + \frac{11k^2}{48}\right]$$

A/2010

Q7(b) Solve the Difference Equation

$$y_{k+2} - 7y_{k+1} + 12y_k = \cos k \quad \text{---} (1)$$

Solution :- For C-F

$$y_{k+2} - 7y_{k+1} + 12y_k = 0$$

Put $y_k = Ab^k$

So $Ab^{k+2} - 7Ab^{k+1} + 12Ab^k = 0$

$$Ab^k[b^2 - 7b + 12] = 0$$

Since $Ab^k \neq 0$ So $b^2 - 7b + 12 = 0$

$$b^2 - 3b - 4b + 12 = 0$$

$$b(b-3) - 4(b-3) = 0$$

$$(b-3)(b-4) = 0$$

$$b = 3, 4$$

Roots are real and unequal

Hence Complementary Solution is

$$y_c = C_1(3)^k + C_2(4)^k$$

Now we find P-I

Put $y_k = C_3 \sin k + C_4 \cos k$

Equation (1) becomes

$$[C_3 \sin(k+2) + C_4 \cos(k+2)] - 7[C_3 \sin(k+1) + C_4 \cos(k+1)] + 12[C_3 \sin k + C_4 \cos k] = \cos k$$

Solved by
Labeeb Ahmad

$$\left[C_3(\sin R\cdot\cos 2+\cos R\cdot\sin 2)+C_4(\cos R\cdot\cos 2-\sin R\cdot\sin 2)\right.$$

$$-7\left[C_3(\sin R\cdot\cos 1+\cos R\cdot\sin 1)+C_4(\cos R\cdot\cos 1-\sin R\cdot\sin 1)\right]$$

OR

$$+12\left[C_3\sin R+C_4\cos R\right]=\cos R$$

$$C_3\sin R\cdot\cos 2+C_3\cos R\cdot\sin 2+C_4\cos R\cdot\cos 2$$
$$-C_4\sin R\cdot\sin 2-7C_3\sin R\cdot\cos 1-7C_3\cos R\cdot\sin 1$$
$$-7C_4\cos R\cos 1+7C_4\sin R\cdot\sin 1+12C_3\sin R$$
$$+12C_4\cos R=\cos R$$

$$\left(C_3\cos 2-C_4\sin 2-7C_3\cos 1+7C_4\sin 1+12C_3\right)\sin R+$$
$$\left(C_3\sin 2+C_4\cos 2-7C_3\sin 1-7C_4\cos 1+12C_4\right)\cos R=\cos R$$

Comparing the Co-efficient of $\sin R$ and $\cos R$

$$C_3\cos 2-7C_3\cos 1+12C_3-C_4\sin 2+7C_4\sin 1=0$$

$$C_3\sin 2-7C_3\sin 1+C_4\cos 2-7C_4\cos 1+12C_4=1$$

$$(\cos 2-7\cos 1+12)C_3-(\sin 2-7\sin 1)C_4=0$$

$$(\sin 2-7\sin 1)C_3+(\cos 2-7\cos 1+12)C_4=1$$

let $\cos 2-7\cos 1+12=l_1$

And $\sin 2-7\sin 1=l_2$

So

$\frac{C_3}{l_2}=$

and

$\frac{C_4}{l_1}$

Hence

$J_R=$

$J_R=$

Solved by
Labeeb Ahm

$$l_1 C_3 - l_2 C_4 = 0$$

$$l_2 C_3 + l_1 C_4 = 1$$

OR

$$l_1 C_3 - l_2 C_4 - 0 = 0$$

$$l_2 C_3 + l_1 C_4 - 1 = 0$$

$$\frac{C_3}{l_2 - 0} = \frac{C_4}{0 + l_1} = \frac{1}{l_1^2 + l_2^2}$$

So

$$\frac{C_3}{l_2} = \frac{1}{l_1^2 + l_2^2} \Rightarrow C_3 = \frac{l_2}{l_1^2 + l_2^2}$$

and

$$\frac{C_4}{l_1} = \frac{1}{l_1^2 + l_2^2} \Rightarrow C_4 = \frac{l_1}{l_1^2 + l_2^2}$$

Solved by
Labeeb Ahmad

Hence the solution (Particular integral)

$$y_k = \frac{l_2}{l_1^2 + l_2^2} \sin k + \frac{l_1}{l_1^2 + l_2^2} \cos k$$

Hence the General solution is

$$y_k = C.F + P.I$$

$$y_k = C_1 (3)^k + C_2 (4)^k + \left[\frac{l_2}{l_1^2 + l_2^2} \sin k + \frac{l_1}{l_1^2 + l_2^2} \cos k \right]$$

where $l_1 = \cos 2 - 7 \cos 1 + 12$

$l_2 = \sin 2 - 7 \sin 1$

A|2010

Q8(a) Solve the system

$$u_{n+1} + u_n - 3v_n = 2^n \quad \text{---} (1)$$

$$2v_{n+1} - v_n + 4u_n = n^2 \quad \text{---} (2)$$

Solution :- Using shift Operator we get

$$(E+1)u_n - 3v_n = 2^n \quad \text{---} (3)$$

$$(2E-1)v_n + 4u_n = n^2 \quad \text{---} (4)$$

Equation (3) multiply by "4" and Equation (4) multiply by $(E+1)$ Then Subtracting :-

$$4(E+1)u_n - 12v_n = 4\cdot 2^n$$

$$-4(E+1)u_n \pm (2E-1)(E+1)v_n = \pm(E+1)n^2$$

$$-12v_n - (2E^2+E-1)v_n = 4\cdot 2^n - [1+\Delta+1]n^2$$

$$-12v_n - 2E^2v_n - Ev_n + v_n = 4\cdot 2^n - [2+\Delta]n^2$$

$$-12v_n - 2v_{n+2} - v_{n+1} + v_n = 4\cdot 2^n - [2n^2 + \Delta n^2]$$

$$-2v_{n+2} - v_{n+1} - 11v_n = 4\cdot 2^n - [2n^2 + (n+1)^2 - n^2]$$

$$-[2v_{n+2} + v_{n+1} + 11v_n] = -[4\cdot 2^n + ((n+1)^2 + n^2)]$$

Solved by
Labeeb Ahmad

$$2\gamma_{n+2} + \gamma_{n+1} + 11\gamma_n = -4\cdot 2^n + [n^2+2n+1+n^2]$$

$$= -4\cdot 2^n + [2n^2+2n+1] \quad (6)$$

For C-F

$$\therefore 2\gamma_{n+2} + \gamma_{n+1} + 11\gamma_n = 0$$

put $\gamma_n = Ab^n$

$$2Ab^{n+2} + Ab^{n+1} + 11Ab^n = 0$$

$$Ab^n [2b^2 + b + 11] = 0$$

Since $Ab^n \neq 0$, So $2b^2 + b + 11 = 0$

using Quadratic formula

$$b = \frac{-1 \pm \sqrt{1 - 4(2)(11)}}{2(2)}$$

$$b = \frac{-1 \pm \sqrt{1 - 88}}{4}$$

$$b = \frac{-1 \pm \sqrt{-87}}{4}$$

$$b = \frac{-1 \pm i\sqrt{87}}{4}$$

$$R = \sqrt{x^2+y^2} = \sqrt{\left[\frac{-1}{4}\right]^2 + \left[\frac{\sqrt{87}}{4}\right]^2}$$

$$= \sqrt{\frac{1}{4} + \frac{87}{4}} = \sqrt{\frac{88}{4}} = \sqrt{22}$$

Solved by Labeeb Ahmad

$$\theta = \tan^{-1}(y/x) = \tan^{-1}\left[\frac{\sqrt{87}/4}{-1/4}\right] = \tan^{-1}(-\sqrt{87})$$

And $\theta = \tan^{-1}(y/x) = \left[\frac{+\sqrt{87}/4}{+1/4}\right] = \tan^{-1}\sqrt{87}$

Hence the Complementry solution is

$$\gamma_n = (\sqrt{22})^n \left[A\cos n\left\{\tan^{-1}(-\sqrt{87})\right\} + iB\sin n\left\{\tan^{-1}(-\sqrt{87})\right\}\right]$$

$$+ (\sqrt{22})^n \left[C\cdot\cos n\left\{\tan^{-1}\sqrt{87}\right\} + iD\sin n\left\{\tan^{-1}\sqrt{87}\right\}\right]$$

Now we find P-I for $-4\cdot 2^n$

$$2\gamma_{n+2} + \gamma_{n+1} + 11\gamma_n = -4\cdot 2^n$$

Put $\gamma_n = C_1 2^n$

$$2C_1\cdot 2^{n+2} + C_1\cdot 2^{n+1} + 11\cdot C_1\cdot 2^n = -4\cdot 2^n$$

$$2^n\left[8C_1 + 2C_1 + 11C_1\right] = -4\cdot 2^n$$

$$21C_1 = -4$$

$$\boxed{C_1 = -4/21}$$

So $\gamma_n = \frac{-4}{21}\cdot 2^n$

Now for Pu

Equa

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$+(n$

$2[C_{2+}$

$+(n$

"

Compa

$2C_4$

Comp

$2C_3$

$14C_3$

Solved by
Labeeb Ahmad

$^{-1}(-\sqrt{87})$

Tan$^{-1}\sqrt{87}$

Sin $n\{$Tan^{-1}

$\{$Tan$^{-1}\sqrt{87}\}$

4.2^n //

2^n

y

nad

Now we find Particular integral for $2n^2+2n+1$

Put $\gamma_n = C_2 + nC_3 + n^2C_4$

Equation (6) becomes

Solution is $2[C_2+(n+2)C_3+(n+2)^2C_4]+[C_2+(n+1)C_3+(n+1)^2C_4]+11[C_2+nC_3+n^2C_4]=2n^2+2n+$

$2[C_2+(n+2)C_3+(n^2+4n+4)C_4]+[C_2+(n+1)C_3+(n^2+2n+1)C_4]+11[C_2+nC_3+n^2C_4]=2n^2+2n+$

Comparing the Co-efficient of "n^2"

$2C_4 + C_4 + 11C_4 = 2$

$14C_4 = 2$

$7C_4 = 1$

$\boxed{C_4 = 1/7}$

Solved by Labeeb Ahmad

Compare the Co-efficient of "n"

$2C_3 + 8C_4 + C_3 + 2C_4 + 11C_3 = 2$

$14C_3 + 10C_4 = 2$

$$14C_3 + 10\left[\frac{1}{7}\right] = 2$$

$$14C_3 = 2 - \frac{10}{7} = \frac{4}{7}$$

$$C_3 = \frac{1}{14} \times \frac{4}{7} \Rightarrow \boxed{\frac{2}{49} = C_3}$$

Now Compare Constant

$$2C_2 + 4C_3 + 8C_4 + C_2 + C_3 + C_4 + 11C_2 = 1$$

$$14C_2 + 5C_3 + 9C_4 = 1$$

$$14C_2 + 5\left(\frac{2}{49}\right) + 9\left(\frac{1}{7}\right) = 1$$

$$14C_2 + \frac{10}{49} + \frac{9}{7} = 1$$

$$14C_2 = 1 - \frac{9}{7} - \frac{10}{49} = -\frac{24}{49}$$

$$C_2 = \frac{1}{14} \times \frac{-24}{49} \Rightarrow \boxed{C_2 = -\frac{12}{343}}$$

Hence P.I is $a_n = \frac{n^2}{7} + \frac{2n}{49} - \frac{12}{343}$

Thus The General Solution is $a_n = C.F + P.I$

$$a_n = (\sqrt{22})^n \left[A \cos n\{\tan^{-1}(-\sqrt{87})\} + iB \sin n\{\tan^{-1}(-\sqrt{87})\}\right.$$

$$+ (\sqrt{22})^n \left[C \cdot \cos n\{\tan^{-1}(\sqrt{87})\} + iD \sin \cdot n\{\tan^{-1}(\sqrt{87})\}\right.$$

$$- \frac{4}{21} 2^n + \left[\frac{n^2}{7} + \frac{2n}{49} - \frac{12}{343}\right]$$

Solved by
Labeeb Ahmad

A/2009

Q7(b) Solve the Difference equation

$$y_{R+2} - 3y_{R+1} + 2y_R = k^2 \quad \text{---} (1)$$

Solution :- For C.F

$$y_{R+2} - 3y_{R+1} + 2y_R = 0$$

put $y_R = Ab^R$

So $Ab^{R+2} - 3Ab^{R+1} + 2Ab^R = 0$

$$Ab^R[b^2 - 3b + 2] = 0$$

Since $Ab^R \neq 0$ Then for

$$b^2 - 3b + 2 = 0$$

$$b^2 - b - 2b + 2 = 0$$

$$b(b-1) - 2(b-1) = 0$$

$$(b-1)(b-2) = 0$$

$$b = 2, 1$$

Roots are real and unequal So Complementry Solution is

$$y_c = c_1(1)^R + c_2(2)^R$$

Now we find P.I

put $y_R = kC_3 + C_4k^2 + C_5k^3$ in (1)

$$[(k+2)C_3 + (k+2)^2C_4 + (k+2)^3C_5] - 3[(k+1) + (k+1)^2C_4 + (k+1)^3C_5] + 2[kC_3 + k^2C_4 + k^3C_5] = k^2$$

Solved by Labeeb Ahmad

$$[(k+2)C_3 + (k^2+4k+4)C_4 + (k^3+6k^2+12k+8)C_5]$$
$$-3[(k+1)C_3 + (k^2+2k+1)C_4 + (k^3+3k^2+3k+1)C_5]$$
$$+2[kC_3 + k^2C_4 + k^3C_5] = k^2$$

Now Compare the Co-efficient of "k^2"

$$C_4 + 6C_5 - 3C_4 = 9C_5 + 2C_4 = 1$$

$$-3C_5 = 1$$

$$\boxed{C_5 = -1/3}$$

Now Compare the Co-efficient of "k"

$$C_3 + 4C_4 + 12C_5 - 3C_3 - 6C_4 - 9C_5 + 2C_3 = 0$$

$$-2C_4 + 3C_5 = 0$$

$$-2C_4 + 3\left(\frac{-1}{3}\right) = 0$$

$$-2C_4 - 1 = 0$$

$$\boxed{C_4 = -1/2}$$

Now Compare Constant

$$2C_3 + 4C_4 + 8C_5 - 3C_3 - 3C_4 - 3C_5 = 0$$

$$-C_3 + C_4 + 5C_5 = 0$$

$$-C_3 + \left(\frac{-1}{2}\right) + 5\left(\frac{-1}{3}\right) = 0$$

$$-C_3 = \frac{1}{2} + \frac{5}{3} \Rightarrow \boxed{C_3 = -\frac{13}{6}}$$

So

$y_k =$

Hence

$y_k =$

$y_k = C$

Solved by: Labeeb Ahmad

Put

there for

Since

$k+8)C_5$ So P-I is

$3k+1)C_5$ $y_k = -\frac{k^3}{3} - \frac{k^2}{2} - \frac{13k}{6}$

Hence General Solution is

of "k" $y_k = C.F + P.I$

$= 1$

$$y_k = C_1(1)^k + C_2(2)^k - \left[\frac{k^3}{3} + \frac{k^2}{2} + \frac{13}{6}k\right]$$

S/2009, A/2000, A/2002

Q7(a) Solve the Difference Equation

$$y_{n+2} - 9y_{n+1} + 20y_n = 4^n(n^2+1) \quad \text{---(1)}$$

of "k"

$C_5 + 2C_3 = 0$

lved by
b Ahmad

Solution:- For C-F

$$y_{n+2} - 9y_{n+1} + 20y_n = 0$$

Put $y_n = Ab^n$

There for $Ab^{n+2} - 9Ab^{n+1} + 20Ab^n = 0$

$$Ab^n[b^2 - 9b + 20] = 0$$

$C_5 = 0$

Since $Ab^n \neq 0$ There for $b^2 - 9b + 20 = 0$

$$b^2 - 4b - 5b + 20 = 0$$

$$b(b-4) - 5(b-4) = 0$$

$$(b-4)(b-5) = 0$$

$$b = 4, 5$$

Solved by
Labeeb Ahmad

roots are real and unequal

So the Complementry solution is

$y_c = C_1(4)^n + C_2(5)^n$

Now we find P-I

Put $y_n = 4^n[nC_3 + n^2C_4 + n^3C_5]$

Equ (1) becomes

$4^{n+2}[(n+2)C_3 + (n+2)^2C_4 + (n+2)^3C_5] - 9\cdot 4^{n+1}$

$[(n+1)C_3 + (n+1)^2C_4 + (n+1)^3C_5] + 20[nC_3 + n^2C_4 + n^3C_4] = 4^n(n^2+1)$

$4^n\{16[(n+2)C_3 + (n^2+4n+4)C_4 + (n^3+6n^2+12n+8)C_5]$

$-36[(n+1)C_3 + (n^2+2n+1)C_4 + (n^3+3n^2+3n+1)C_5]$

$+20[nC_3 + n^2C_4 + n^3C_5] = 4^n(n^2+1)$

Now Comparing the Co-efficient "n^2"

$16C_4 + 96C_5 - 36C_4 - 108C_5 + 20C_4 = 1$

$-12C_5 = 1$

$\boxed{C_5 = -1/12}$

Now

$16C_3 \cdot 7$

Now

$32C_3 \cdot$

$-4C$

$= 4C_3$

$-4.$

-4

So P=

Hence

$= C_1$

Solved by
Labeeb Ahmad

Now Compare the Co-efficient of "n"

$$16C_3 + 64C_4 + 192C_5 - 36C_3 - 72C_4 - 108C_5 + 20C_3 = 0$$

$$-8C_4 + 84C_5 = 0$$

$$-8C_4 + 84\left(\frac{-1}{12}\right) = 0$$

$$-8C_4 - 7 = 0$$

$$\boxed{C_4 = -7/8}$$

Now Compare Constant

$$32C_3 + 64C_4 + 128C_5 - 36C_3 - 36C_4 - 36C_5 = 1$$

$$-4C_3 + 28C_4 + 92C_5 = 1$$

$$-4C_3 + 28\left(\frac{-7}{8}\right) + 92\left(\frac{-1}{12}\right) = 1$$

$$-4C_3 - \frac{49}{2} - \frac{23}{3} = 1$$

$$-4C_3 = 1 + \frac{49}{2} + \frac{23}{3} = \frac{199}{6}$$

$$\boxed{C_3 = -\frac{199}{24}}$$

Solved by
Labeeb Ahmad

So P-I is $y_n = 4^n\left[-\frac{n^3}{12} - \frac{7n^2}{8} - \frac{199}{24}n\right]$

Hence General solution is

$$y_n = C_1(4)^n + C_2(5)^n - 4^n\left[\frac{n^3}{12} + \frac{7n^2}{8} + \frac{199}{24}n\right]$$

5/2009, A/2000

Q7(b) Solve the Difference Equation

$$y_{n+2} - 2y_{n+1} + y_n = \sin(5n) + \cos 5n + 6 \quad \text{---(1)}$$

Solution:- For C-F

$$y_{n+2} - 2y_{n+1} + y_n = 0$$

Put $y_n = Ab^n$

So $Ab^{n+2} - 2Ab^{n+1} + Ab^n = 0$

$$Ab^n[b^2 - 2b + 1] = 0$$

Since $Ab^n \neq 0$ Therefor $b^2 - 2b + 1 = 0$

$$b^2 - b - b + 1 = 0$$

$$b(b-1) - 1(b-1) = 0$$

$$(b-1)(b-1) = 0$$

$$b = 1, 1$$

Solved by
Labeeb Ahmad

Roots are real and equal, So Complementry solution is

$$y = (C_1 + nC_2)(1)^n$$

Now we find P-I seperately

P-I for "6"

Put $y_n = n^2 C$

Equation (i) Takes form

$$(n+2)^2 C - 2(n+1)^2 C + n^2 C = 6$$

$(n^2 +$

$n^2 C +$

Solv

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So e

$[C_1 \sin$

$C_2 \cos 5(n +$

$\Rightarrow$

$[C_1 \sin$

$C_2 \cos(5$

$C_1(\sin 5n$

$2C_2(\sin 5r$

+

$C_1 \sin 5n$

$2C_2 \sin 5n \cdot 6$

+

Equation $(n^2+4n+4)C - 2(n^2+2n+1)C + n^2C = 6$

$n^2C + 4nC + 4C - 2n^2C - 4nC - 2C + n^2C = 6$

$4C - 2C = 6$

Solved by Labeeb Ahmad

$4C = 6$

$C = \frac{6}{4} \Rightarrow \boxed{C = \frac{3}{2}}$

So $y_n = \frac{3}{2}n^2$

Now P-I for $\sin(5n) + \cos 5n$

put $y_n = C_1 \sin 5n + C_2 \cos 5n$

So equation (i) becomes

$[C_1 \sin 5(n+2) + C_2 \cos 5(n+2)] - 2[C_1 \sin 5(n+1) + C_2 \cos 5(n+1)] + [C_1 \sin 5n + C_2 \cos 5n] = \sin 5n + \cos$

$\Rightarrow$

$[C_1 \sin(5n+10) + C_2 \cos(5n+10)] - 2[C_1 \sin(5n+5) + C_2 \cos(5n+5)] + [C_1 \sin 5n + C_2 \cos 5n] = \sin 5n + \cos 5$

separately

$C_1(\sin 5n \cdot \cos 10 + \cos 5n \cdot \sin 10) + C_2(\cos 5n \cdot \cos 10 - \sin 5n \sin 10)$

$-2C_2(\sin 5n \cdot \cos 5 + \cos 5n \cdot \cos 5) - 2C_4(\cos 5n \cdot \cos 5 - \sin 5n \cdot \sin 5)$

$+ C_1 \sin 5n + C_2 \cos 5n = \sin 5n + \cos 5n$

$\Rightarrow C_1 \sin 5n \cdot \cos 10 + C_1 \cos 5n \cdot \sin 10 + C_2 \cos 5n \cdot \cos 10 - C_2 \sin 5n \sin$

$-2C_2 \sin 5n \cdot \cos 5 - 2C_2 \cos 5n \cdot \cos 5 - 2C_4 \cos 5n \cdot \cos 5 + 2C_4 \sin 5n \cdot \sin$

$+ C_1 \sin 5n + C_2 \cos 5n = \sin 5n + \cos 5n$

$[C_1 \cos 10 - C_2 \sin 10 - 2C_1 \cos 5 + 2C_2 \sin 5 + C_1] \sin 5n$

$-[C_1 \sin 10 + C_2 \cos 10 - 2C_1 \sin 5 - 2C_2 \cos 5 + C_2] \cos 5n$

$= \sin 5n + \cos 5n$

Comparing the co-efficient of $\sin 5n$ & $\cos 5n$

$C_1 \cos 10 - C_2 \sin 10 - 2C_1 \cos 5 + 2C_2 \sin 5 + C_1 = 1 \quad -(1)$

$C_1 \sin 10 + C_2 \cos 10 - 2C_1 \sin 5 - 2C_2 \cos 5 + C_2 = 1 \quad -(2)$

Equation (1) & (2) can be re-arrange

$(\cos 10 - 2\cos 5 + 1)C_1 + (-\sin 10 + 2\sin 5)C_2 = 1 \quad -(3)$

$-(-\sin 10 + 2\sin 5)C_1 + (\cos 10 - 2\cos 5 + 1)C_2 = 1 \quad -(4)$

Let $\cos 10 - 2\cos 5 + 1 = l_1$

$-\sin 10 + 2\sin 5 = l_2$

Equation (3) & (4) takes the form

$l_1 C_1 + l_2 C_2 = 1$

$-l_2 C_1 + l_1 C_2 = 1$

OR

$l_1 C_1 + l_2 C_2 - 1 = 0$

$-l_2 C_1 + l_1 C_2 - 1 = 0$

C_1

$-l_2 + l$

So C

$l_1 -$

$\Rightarrow$

C_1

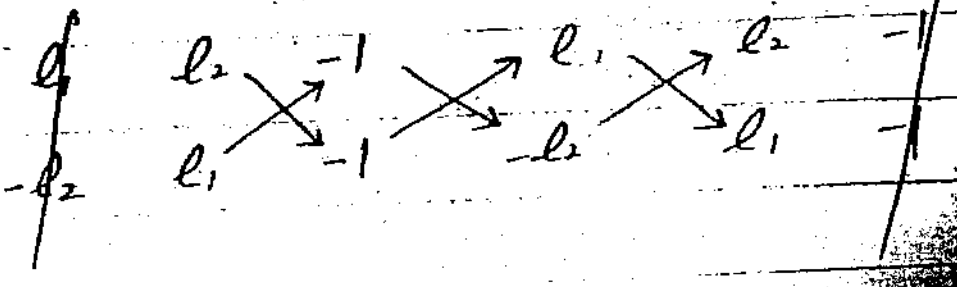
So P.I

Hence

$y_n = ($

$+($

Solved by
Labeeb Ahmad



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$$\frac{C_1}{-l_2+l_1} = \frac{C_2}{l_2+l_1} = \frac{1}{l_1^2+l_2^2}$$

So

$$\frac{C_1}{l_1-l_2} = \frac{1}{l_1^2+l_2^2} \quad \text{and} \quad \frac{C_2}{l_1+l_2} = \frac{1}{l_1^2+l_2^2}$$

$$\Rightarrow \quad C_1 = \frac{l_1-l_2}{l_1^2+l_2^2} \quad \text{and} \quad C_2 = \frac{l_1+l_2}{l_1^2+l_2^2}$$

So P.I is $y_n = \frac{l_1-l_2}{l_1^2+l_2^2}\sin 5n + \frac{l_1+l_2}{l_1^2+l_2^2}\cos 5n$

Hence the General Solution is

$$y_n = (C_1+C_2 n)(1)^n + \frac{3}{2}n^2 + \left(\frac{l_1-l_2}{l_1^2+l_2^2}\right)\sin 5n + \left(\frac{l_1+l_2}{l_1^2+l_2^2}\right)\cos 5n$$

where $l_1 = \cos 10 - 2\cos 5 + 1$

$l_2 = -\sin 10 + 2\sin 5$

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From Sharaqpur
0341-7224005

P

Q2(

method

Solutio

$y_1 = f(x_0$

Also

y_1

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That i

$y_1 = y$

Now E

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$y_1 = y_0 +$

$= y_0 +$

$= y_0 + h$

PAPER S/2015

Q2(a) Desive the Rung-Kutta method of order two.

Solution {S/2011}{S/2009}{A/1999}

As we know

$$y_1 = y(x_0+h) = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \text{---------} \quad (\text{BJ - Taylor series})$$

Also Improved Euler method gives

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

where $x_1 = x_0 + h$

and $y_1^{(1)} = y_0 + hf(x_0, y_0)$

That is

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0+h,\ y_0 + hf(x_0, y_0))\right]$$

Now Expanding $f(x_0+h,\ y_0+hf(x_0, y_0))$

by Taylor series

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0, y_0) + h\left(\frac{\partial f}{\partial x}\right)_{x_0, y_0} \cdot 1 + hf(x_0, y_0)\left(\frac{\partial f}{\partial y}\right)_{x_0, y_0} + \cdots\cdots\right]$$

$$y_1 = y_0 + \frac{h}{2}\left[2f(x_0, y_0) + h\left\{\left(\frac{\partial f}{\partial x}\right)_{x_0, y_0} + f(x_0, y_0)\left(\frac{\partial f}{\partial y}\right)_{x_0, y_0}\right\}\right.$$

$$y_1 = y_0 + hf(x_0, y_0) + \frac{h^2}{2!}f'(x_0, y_0) + \text{---------}$$

Solved by
Laheeb Ahmad

where $f'(x_0, y_0) = \left[\left(\frac{\partial f}{\partial x}\right)_{x_0, y_0} + f(x_0, y_0)\left(\frac{\partial f}{\partial y}\right)\right]$

Hence $y_0 = y_0 + h f(x_0, y_0) + \frac{h^2}{2!} f'(x_0, y_0) + \cdots$ ——— (B)

Equation (A) and (B) imply that improved Euler's method agree with Taylor's Series section upto the term in h^2.

Here for the improved Euler's meked is Rung Kutta method of second order.

PAPER A12014

Q4(a) Using Runge-Kutta method order fourth; solve for $y(0.1)$ given that

$\frac{dy}{dx} = x + y$, $y(0) = 0$, $h = 0.05$

Solution

$y_{n+1} = y_n + K$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

Given $y' = x + y$; $y(0) = 0$

$x_0 = 0$, $y_0 = 0$

$x_1 = 0.05$, $h = 0.05$

$f(x_n, y_n) = x_n + y_n$

$f(x_0, y_0) = x_0 + y_0 = 0 + 0 = 0$

Solved by

$\frac{dy}{dx}$

$K_1 = h f(x_0, y_0)$

$= (0.05)(0)$

$= 0$

Solved by Labeeb Ahmad

$K_2 = h f\left[x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right]$

$= (0.05) f\left[0 + \frac{0.05}{2}, 0 + \frac{0}{2}\right]$

$= (0.05) f(0.025, 0)$

$= (0.05)(0.025 + 0)$

$K_2 = 0.00125$

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.05) f\left[0 + \frac{0.05}{2}, 0 + \frac{0.00125}{2}\right]$

$= (0.05) f(0.025, 0.000625)$

$= (0.05)(0.025 + 0.000625)$

$= 0.00128125$

$K_4 = h f(x_0 + h, y_0 + K_3)$

$= (0.05) f(0 + (0.05), 0 + 0.00128125)$

$= (0.05) f(0.05, 0.00128125)$

$= 0.0025640$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0 + 2(0.00125) + 2(0.00128125) + 0.0025640)$

$K = 0.0012710$

$y_1 = y_0 + K \Rightarrow y_1 = 0 + 0.0012710$

$y_1 = 0.0012710$

Now $K_1 = hf(x_1, y_1)$

$= (0.05) f(0.05, 0.0012710)$

$= (0.05)[0.05 + 0.0012710)$

$K_1 = 0.0025635$

$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$= (0.05) f\left[0.05 + \frac{0.05}{2}, 0.0125 + \frac{0.0025635}{2}\right]$

$= (0.05) f(0.075, 0.00255)$

$= (0.05)(0.075 + 0.00255)$

$K_2 = 0.0032775$

$K_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$

$= (0.05) f\left(0.05 + \frac{0.05}{2}, 0.00127 + \frac{0.0032775}{2}\right)$

$= (0.05)(0.075 + 0.0032087)$

$K_3 = 0.0039043$

$K_4 = hf(x_1 + h, y_1 + K_3)$

$= (0.05) f(0.05 + 0.05, 0.00127 + 0.0039043)$

$= (0.05) f(0.1, 0.005180)$

$= (0.05)[0.1 + 0.005180]$

$K_4 = 0.0052590$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.0025635 + 2(0.0032775) + 2(0.0039043)$

Solved by
Labeeb Ahm

$K = 0.003895$

$y_2 = y_1 + K$

$y_2 = 0.0012710 + 0.003895$

$y_2 = 0.0051705$

Now $\Rightarrow$ $K_1 = hf(x_2, y_2)$

$K_1 = h[x_2 + y_2]$

$K_1 = (0.05)[0.1 + 0.0051705]$

$K_1 = 0.005258$

$K_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2}\right)$

$= (0.05) f\left(0.1 + \frac{0.05}{2}, 0.0051705 + \frac{0.005258}{2}\right)$

$= (0.05)[0.125 + 0.0077996]$

$K_2 = 0.0066399888$

$K_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2}\right)$

$= (0.05) f\left(0.125, 0.0051705 + \frac{0.0066399888}{2}\right)$

$= (0.05)[0.125 + 0.0084905]$

$K_3 = 0.0066745$

$K_4 = hf(x_2 + h, y_2 + K_3)$

$= (0.05) f(0.1 + 0.05, 0.0051705 + \frac{0.006674}{\ }$

$= (0.05)[0.15 + 0.0118445]$

$K_4 = 0.0080922$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$K = \frac{1}{6}[0.005258 + 2(0.006639) + 2(0.006674) + 0.0080922]$

$K = 0.0066627$

$y_3 = y_2 + K \Rightarrow 0.0051705 + 0.0066627 = 0.0118332$

Solved by
Labeeb Ahmad

Q5(b) A|2014

Write an algorithem to Solve the D-E $y' = -2xy$, $y(0) = 1$ at $x = 0.1$ using Runge-Kutta method of Second order

Solution :- The algorithem to Runge-Kutta method of 2nd order is

$K_1 = hf(x_0, y_0)$ ..... $K_2 = hf(x_0+h, y_0+K_1)$

$K = \frac{1}{2}(K_1 + K_2)$

And $y_{n+1} = y_n + K$

Given :- $f(x_0, y_n) = -2x_n y_n$

$x_0 = 0$, $y_0 = 1$, $h = 0.1$

$f(x_0, y_0) = -2x_0 \cdot y_0 = -2(0)(1) = 0$

$K_1 = hf(x_0, y_0) = (0.1)(0) = 0$

$K_2 = hf(x_0+h, y_0+K_1) = (0.1) f(0+0.1, 1)$

$= (0.1)[-2(0.1)(1)] = -0.02$

$K = \frac{1}{2}[K_1 + K_2] = \frac{1}{2}[0 - 0.02] = -0.01$

$y_1 = y_0 + K$

$= 1 + (-0.01)$

$y_1 = 0.99$

Solved by
Labeeb Ahmad

$K_1 = h f(x_1, y_1)$

$= (0.1) f(0.1, 0.99)$

$= (0.1)[-2(0.1)(0.99)]$

$K_1 = -0.0198$

$K_2 = h f(x_1 + h, y_1 + K_1)$

$= (0.1) f((0.1)+(0.1), (0.99)+(-0.0198))$

$K_2 = (0.1) f(0.2, -0.019602)$

$= (0.1)[-2(0.2)(-0.019602)]$

$K_2 = 0.00078408$

$K = \frac{1}{2}[K_1 + K_2]$

$= \frac{1}{2}[-0.0198 + 0.00078408]$

$K = -0.00950796$

$y_2 = y_1 + K$

$y_2 = 0.99 + (-0.00950796)$

$y_2 = 0.98049204$

Solved by:
Labeeb Ahmad

PAPER A/2013

Q3(b)

Solved in Sir Ajmal (Notes)

PAPER A/2012

Q7(b) Solve the D-E given below with the help of 4th-R-K method, by taking $h = 0.2$

$$y' = 2ty^2$$

$$y(0) = 1 \quad 0 \le t \le 0.4$$

Solution Given $y' = 2ty^2$

Replace t by x

$$y' = 2xy^2$$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$

$f(x_n, y_n) = 2x_n y_n^2$

$f(x_0, y_0) = 2x_0 \cdot y_0^2$

$= 2(0)\cdot(1)^2 = 0$

$K_1 = hf(x_0, y_0)$

$= (0.2)(0) = 0$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0}{2}\right)$

$= (0.2) f(0.1, 1)$

$= 0.04$

Solved by
Labeeb Ahmad

$$K_3 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_2}{2}\right)$$

$$K_3 = (0.2) f\left(0 + \frac{0.2}{2},\ 1 + \frac{0.04}{2}\right)$$

given — R-K

$$= 0.02\,(0.2) f(0.1,\ 1.02)$$

$$= (0.2)(0.20808)$$

$$K_3 = 0.041616$$

$$K_4 = hf(x_0 + h,\ y_0 + K_3)$$

$$= (0.2) f(0 + 0.2,\ 1 + 0.041616)$$

x

$$= (0.2) f(0.2,\ 1.041616)$$

$$= (0.2)(0.43398)$$

0.2

$$K_4 = 0.08679$$

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K = \frac{1}{6}\left(0 + 2(0.04) + 2(0.041616) + (0.08679)\right]$$

$$K = 0.041670333$$

$$y_1 = y_0 + K$$

$$y_1 = 1 + 0.041670333$$

$$y_1 = 1.041670333$$

Now

Solved by
Labeeb Ahmad

$K_1 = h f(x_1, y_1)$

$= (0.2) f(0.2, 1.041670)$

$= (0.2)(0.434030)$

$K_1 = 0.08680$

$K_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$= (0.2) f\left(0.2 + \frac{0.2}{2}, 1.041670 + \frac{0.08680}{2}\right)$

$= (0.2) f(0.3, 1.08507)$

$= (0.2)(0.706426)$

$K_2 = 0.141285$

$K_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$

$K_3 = (0.2) f\left(0.2 + \frac{0.2}{2}, 1.041670 + \frac{0.14128}{2}\right)$

$= (0.2) f(0.3, 1.112312)$

$= (0.2)(0.74234)$

$K_3 = 0.14846$

$K_4 = h f(x_1 + h, y_1 + K_3)$

$= (0.2) f(0.2 + 0.2, 1.041670 + 0.14846)$

$= (0.2) f(0.4, 1.19013)$

$= (0.2)(1.13312)$

$K_4 = 0.226625$

K =

K =

K =

y_2 =

y_2 =

y_2 =

A|2012

Q

value

Take

with

Solu

Solved by
Labeeb Ahmad

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K = \frac{1}{6}(0.08680 + 2(0.14285) + 2(0.14846) + 0.22625)$$

$$K = 0.148819166$$

$\frac{0.08680}{2}$

$$y_2 = y_1 + K$$

$$y_2 = 1.046703 + 0.148819166$$

$$y_2 = 1.190489$$

$\frac{0.14128}{2}$

$0.14846)$

A|2012

Q9(b)

Solve the following initial value Problem using Taylor's method order:

$y' = 3y + 3t$ with $y(0) = 1$

Take $h = 0.2$ and Compare your solution with the exact solution at $t = 0.4$

Solution :-

Given $y' = 3y + 3t$

Replace "t" by "x"

$y' = 3(y + x)$

$\therefore y(0) = 1$

$x_0 = 0$; $y_0 = 1$ $h = 0.2$

Solved by

Labeeb A...

$y' = 3y + 3x = 3(y_0) + 3(x_0) = 3(1) + 3(0) = 3$

$y'' = 3y' + 3$

$= 3(3) + 3 = 12$

$y''' = 3y'' = 3(12) = 36$

$y^{(iv)} = 3y''' = 3(36) = 108$

By Taylor's Series

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0''' + \cdots$$

$$= 1 + (0.2)(3) + \frac{(0.2)^2}{2}(12) + \frac{(0.2)^3}{3!}(36) + \frac{(0.2)^4}{4!}(108) + \cdots$$

$y_1 = 1 + 0.6 + 0.24 + 0.162 + 0.0072 + \cdots$

$y_1 = 2.0092$

Solved by
Labeeb Ahmad

$y(0.2) = 2.0092$

$x_1 = 0.2 \quad ; y_1 = 2.0092$

Now

$y_1' = 3y_1 + 3x_1 = 3(0.2 + 2.0092)$

$= 6.6276$

$y_1'' = 3y_1' + 3 = 3(6.6276) + 3 = 22.8828$

$y_1''' = 3y_1'' = 3(22.8828) = 68.6484$

$y_1^{(iv)} = 3y_1''' = 3(68.6484) = 205.9452$

By Taylor's Series

$$y_2 = y_1 + hy_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \frac{h^4}{4!}y_1^{(iv)} + \cdots$$

$y_2 = (0.2)$
$y_2 =$
$y_2 =$
$y_2 =$
$y_2' =$
$y_2'' =$
$y_2''' =$
$y_2^{(iv)} =$
By
$y_3 =$
$y_3 =$
(0.2)

$$y_2 = (2.0092) + (0.2)(6.6276) + \frac{(0.2)^2}{2!}(22.88$$

$$+\frac{(0.2)^3}{3!}(68.6484) + \frac{(0.2)^4}{4!}(205.94$$

$$y_2 = 2.0092 + 1.32552 + 0.457656$$

$$+0.0915312 + 0.013729 + \text{——}$$

$$y_2 = 3.897636$$

$$y_2 = y(0.4) = 3.897636$$

$$x_2 = 0.4 \quad , \quad y_2 = 3.897636$$

$$y_2' = 3y_2 + 3x_2 = 3(0.4 + 3.897636)$$

$$= 12.8929$$

$$y_2'' = 3y_2' + 3 = 3(12.8929) + 3$$

$$= 41.6787$$

$$y_2''' = 3y_2'' = 3(41.6787) = 125.0361$$

$$y_2^{(iv)} = 3y_2''' = 3(125.0361) = 375.1085$$

By Taylor's Series

Solved by
Labeeb Ahmad

$$y_3 = y_2 + hy_2' + \frac{h^2}{2!}y_2'' + \frac{h^3}{3!}y_2''' + \frac{h^4}{4!}y_2^{(iv)} +$$

$$y_3 = 3.897636 + (0.2)(12.8929) +$$

$$\frac{(0.2)^2}{2!}(41.6787) + \frac{(0.2)^3}{3!}(125.0361)$$

$$+\frac{(0.2)^4}{4!}(375.1085) + \text{——}$$

$y_3 = 3.897636 + 2.578588 + 0.833574$

$+ 0.1667148 + 0.02500 +$

$y_3 = 7.501512033$

Exact Solution

$$\frac{dy}{dx} = 3y + 3x$$

$$\frac{dy}{dx} - 3y = 3x$$

$I.F = e^{\int -3\,dx} = e^{-3x}$

Multiply by e^{-3x} both side

$$e^{-3x}\cdot\frac{dy}{dx} - 3e^{-3x}y = 3xe^{-3x}$$

$$\frac{d}{dx}(e^{-3x}\cdot y) = 3xe^{-3x}$$

integrating both side

$$e^{-3x}\cdot y = 3\int x e^{-3x}$$

$$e^{-3x}\cdot y = 3\left[x\cdot\frac{e^{-3x}}{-3} - \int\frac{e^{-3x}}{-3}\,dx\right]$$

$$= 3\left[-\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} + c'\right]$$

$$= -xe^{-3x} - \frac{e^{-3x}}{3} + 3c'$$

Solved by Labeeb Ahmad

574

$$e^{-3x}y = -xe^{-3x} - \frac{e^{-3x}}{3} + c \quad \text{(i)}$$

Given $y(0) = 1$ $\quad (\because 3c' = c)$

$$e^{0}(1) = -(0)e^{0} - \frac{e^{0}}{3} + c$$

$$1 = 0 - \frac{1}{3} + c$$

$$1 + \frac{1}{3} = c$$

$$c = \frac{4}{3}$$

put in (i)

$$y = -x - \frac{1}{3} + \frac{4}{3}e^{3x}$$

$$y(0.4) = -(0.4) - \frac{1}{3} + \frac{4}{3}(e^{3(0.4)})$$

$$= 3.693489233$$

Written by
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Mob:0341-7224006

PAPER A/2011

Q5(a) Solve the following difference equation using Modified Euler's method for $y(0.2)$, $y(0.4)$

$$\frac{dy}{dx} = 2xy \quad \text{with} \quad y(0) = 1$$

Solution:-

Formula of Modified Euler method

$$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right]$$

$$= y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2}(2x_n y_n)\right]$$

$$= y_n + hf\left[x_n + \frac{h}{2}, y_n + hx_n y_n\right]$$

$$= y_n + h\left[2\left(x_n + \frac{h}{2}\right)\left(y_n + hx_n \cdot y_n\right)\right]$$

$$= y_n + h\left[(2x_n + h)(y_n + hx_n y_n)\right]$$

$$= y_n + (2hx_n + h^2)(y_n + hx_n y_n)$$

$$y_{n+1} = y_n + \left[2hx_n y_n + 2h^2 x_n^2 y_n + h^2 y_n + h^3 x_n y_n\right]$$

$h = 0.2$

$$y_{n+1} = y_n + [2 \times 0.2 \cdot x_n y_n + 2(0.2)^2 x_n^2 y_n + (0.2)^2 y_n + (0.2)^3 x_n y_n]$$

Solved by
Labeeb Ahmad

y_{n+1}

y_{n+1}

$y_1 =$

$y_1 =$

$y_1 =$

$y_2 =$

$y_2 = ($

$y_2 =$

$y_3 =$

$y_3 =$

$y_3 =$

$$y_{n+1} = y_n + 0.04 x_n y_n + 0.08 x_n^2 y_n$$

$$+ 0.04 y_n + 0.008 x_n y_n$$

$$y_{n+1} = 1.04 y_n + 0.408 x_n y_n + 0.08 x_n^2 y_n$$

difference

Put $n = 0$

zaal

$$y_1 = 1.04 y_0 + 0.408 x_0 y_0 + 0.08 x_0^2 y_0$$

As $y(0) = 1$

=1

$x_0 = 0$; $y_0 = 1$

$$y_1 = 1.04 + 0 + 0$$

$$\boxed{y_1 = 1.04}$$

er method

i]

$$y_2 = 1.04 y_1 + 0.408 x_1 y_1 + 0.08 x_1^2 y_1$$

As $x_1 = 0.2$ and $y_1 = 1.04$

$$y_2 = (1.04)(1.04) + (0.408)(0.2)(1.04)$$

$$+ 0.08 (0.2)^2 (1.04)$$

$$\boxed{y_2 = 1.169792}$$

$$y_3 = (1.04) y_2 + (0.408) x_2 y_2 + 0.08$$

$$x_2^2 y_2$$

As $x_2 = 0.4$; $y_2 = 1.169792$

3 $x_n y_n$]

$$y_3 = (1.04)(1.169792) + (0.408)(0.4)(1.1691$$

$$+ 0.08 (0.4)^2 (1.169792)$$

$y_n +$

$$\boxed{y_3 = 1.422467}$$

n]

Solved by
Labeeb Ahmad

PAPER A/2010

Q9(b) Use the R-K method of 4th order with $h=0.1$ to approximate the solution to

$$\frac{dy}{dx} = \frac{2y}{x^2} + x^2 e^x$$

$$1 \leq x \leq 1.4 \; ; \; y(1) = 0$$

Solution Given

$$f(x,y) = \frac{2y}{x^2} + x^2 e^x$$

$$y(1) = 0$$

$$x_0 = 1 \; ; \; y_0 = 0$$

$$f(x_0, y_0) = \frac{2(0)}{1^2} + (1)^2 e^1 = e$$

$$K_1 = h f(x_0, y_0)$$

$$= (0.1)(e) = 0.2718$$

$$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= (0.1) f\left(1 + \frac{0.1}{2}, 0 + \frac{0.2718}{2}\right)$$

$$= (0.1) f(1.05, 0.1359)$$

$$= (0.1)\left[\frac{2(0.1359)}{(1.05)^2} + (1.05)^2 e^{(1.05)}\right]$$

$$= (0.1)(3.3970)$$

$$K_2 = 0.3397$$

Solved by
Labeeb Ahmad

$K_3 =$

$=$

$=($

$=$

$=($

$K_3 =$

$K_4 =$

$=(0$

$=(0$

$=(0$

$=(0$

$K_4 = 0$

$K =$

$K =$

$K =$

$y_1 =$

$y_1 = ($

$y_1 =$

of 4th

oximate

$)=0$

$(1.05)]$

$$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= (0.1) f\left(1 + \frac{0.1}{2}, 0 + \frac{0.3397}{2}\right)$$

$$= (0.1) f(1.05, 0.16985)$$

$$= (0.1)\left[\frac{2(0.16985)}{(1.05)^2} + (1.05)^2 e^{(1.05)}\right]$$

$$= (0.1)(3.4586)$$

$$K_3 = 0.34586$$

$$K_4 = h f(x_0 + h, y_0 + K_3)$$

$$= (0.1) f(1 + 0.1, 0 + 0.3458)$$

$$= (0.1) f(1.1, 0.3458)$$

$$= (0.1)\left[\frac{2(0.3458)}{(1.1)^2} + (1.1)^2 e^{(1.1)}\right]$$

$$= (0.1)[0.5716 + 3.6350]$$

$$K_4 = 0.42066$$

$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$K = \frac{1}{6}[(0.2718) + 2(0.3397) + 2(0.3458) + (0.42066)]$$

$$K = 0.343910681$$

$$y_1 = y_0 + K$$

$$y_1 = (0) + (0.343910)$$

$$y_1 = 0.343910681$$

Solved by
Labeeb Ahmad

Now $K_1 = h f(x_1, y_1)$

$K_1 = (0.1) f(1.1, 0.3439)$

$= (0.1)\left[\frac{2(0.3439)}{(1.1)^2} + (1.1)^2 e^{(1.1)}\right]$

$= (0.1)(0.5684 + 3.6350)$

$K_1 = 0.42034$

$K_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$K_2 = (0.1) f\left(1.1 + \frac{0.1}{2}, (0.3439) + \frac{0.42034}{2}\right)$

$K_2 = (0.1) f(1.15, 0.55407)$

$K_2 = (0.1)\left[\frac{2(0.55407)}{(1.15)^2} + (1.15)^2 e^{(1.15)}\right]$

$= (0.1)[0.83791 + 4.176710]$

$K_2 = 0.50146$

$K_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$

$= (0.1) f\left(1.1 + \frac{(0.1)}{2}, (0.3439) + \frac{0.50146}{2}\right)$

$= (0.1) f(1.15, 0.59463)$

$= (0.1)\left[\frac{2(0.59463)}{(1.15)^2} + (1.15)^2 . e^{1.15}\right]$

$= (0.1)[0.89925 + 4.176710]$

$K_3 = 0.507596$

$K_4 =$

$K_4 =$

$=$

$=$

$K_4 =$

$K =$

$=$

$K =$

$y_2 =$

$y_2 =$

$y_2 =$

Nou

Solved by
Labeeb Ahmad

$K_4 = h f(x_1 + h, y_1 + K_3)$

$K_4 = (0.1) f((1.1) + (0.1), (0.3439) + (0.507596)$

$= (0.1) f(1.2, 0.851496)$

$= (0.1)\left[\frac{2(0.851496}{(1.2)^2} + (1.2)^2 e^{1.2}\right]$

$= (0.1)[1.1826 + 4.7809]$

$K_4 = 0.596356$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.42034 + 2(0.50146) + 2(0.507$
$+ 0.596356]$

$K = 0.505769$

$y_2 = y_1 + K$

$y_2 = 0.3439 + 0.505769$

$y_2 = 0.84966$

Solved by
Labeeb Ahmad

Now

$K_1 = h f(x_2, y_2)$

$K_1 = (0.1) f(1.2, 0.8496)$

$K_1 = (0.1)\left[\frac{2(0.8496)}{(1.2)^2} + (1.2)^2 e^{(1.2)}\right]$

$= (0.1)(1.18 + 4.78096)$

$K_1 = 0.596096$

$$K_2 = h f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2}\right)$$

$$= (0.1) f\left(1.2 + \frac{0.1}{2}, 0.84966 + \frac{0.596096}{2}\right)$$

$$= (0.1) f(1.25, 1.147708)$$

$$= (0.1)\left[\frac{2(1.147708)}{(1.25)^2} + (1.25)^2 e^{(1.25)}\right]$$

$$= (0.1)(1.4690 + 5.4536)$$

$$K_2 = 0.692266$$

$$K_3 = h f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2}\right)$$

$$= (0.1) f\left(1.2 + \frac{0.1}{2}, 0.84966 + \frac{0.692266}{2}\right)$$

$$= (0.1) f(1.25, 1.19579)$$

$$= (0.1)\left[\frac{2(1.19579)}{(1.25)^2} + (1.25)^2 e^{(1.25)}\right]$$

$$= (0.1)(1.5306 + 5.4536)$$

$$K_3 = 0.69842$$

$$K_4 = h f(x_2 + h, y_2 + K_3)$$

$$= (0.1) f(1.2 + 0.1, 0.84966 + 0.69842)$$

$$= (0.1) f(1.3, 1.54808)$$

$$= (0.1)\left[\frac{2(1.54808)}{(1.3)^2} + (1.3)^2 e^{(1.3)}\right]$$

$$= (0.1)(1.83204 + 6.20111)$$

$$K_4 = 0.803315$$

$K =$

$K =$

$K =$

$y_3 =$

$=$

$y_3 =$

Solved by
Labeeb Ahmad

$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$\frac{0.596096}{2}$)

$$K = \frac{1}{6}[0.596096 + 2(0.692266) + 2(0.69842) + 0.803315$$

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$$K = 0.6959$$

$$y_3 = y_2 + K$$

$$= 0.84966 + 0.6959$$

$$y_3 = 1.545601$$

$\frac{92266}{2}$)

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Solved by
Labeeb Ahmad

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Q9(

$y(0) =$

using

Solu

Sa

K_1

K_2

K_3

$K_3 =$

=

= 1.

= 10

PAPER A12009

Q9(a) Given that $\frac{dy}{dx} = 1+y^2$

$y(0)=0$, $h=0.2$ Find $y(0.4)$

using R-K method order 4.

Solution:- Given $f(x,y) = 1+y^2$

$y(0)=0$

$x_0=0$; $y_0=0$

So $f(x_0,y_0) = 1+y_0^2 = 1+0^2 = 1$

$K_1 = h f(x_0,y_0)$

$= (0.2)(1) = 0.2$

$K_2 = h f\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}\right)$

$= (0.2)\left[f\left(0+\frac{0.2}{2}, 0+\frac{0.2}{2}\right)\right]$

$= (0.2) f(0.1, 0.1)$

$= (0.2)(1+(0.1)^2)$

$K_2 = 0.2\,02$

$K_3 = h f\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right)$

$= (0.2) f\left(0+\frac{0.2}{2}, 0+\frac{0.202}{2}\right)$

$= (0.2) f(0.1, 0.101)$

$= (0.2)(1+(0.101)^2)$

Solved by
Labeeb Ahmad

$K_3 = 0.2020402$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= (0.2) f(0 + 0.2, 0 + 0.2020402)$

$= (0.2) f(0.2, 0.2020402)$

$= (0.2)(1 + (0.2020402)^2)$

$K_4 = 0.208164$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[(0.2) + 2(0.202) + 2(0.2020402) + 0.208164]$

$K = 0.2027074$

$y_1 = y_0 + K$

$y_1 = 0 + 0.2027074$

$\boxed{y_1 = 0.2027074}$ OR $y(0.2) = 0.2027074$

Now:

$K_1 = hf(x_1, y_1)$

$= (0.2) f(0.2, 0.2027)$

$= (0.2)(1 + (0.2027)^2)$

$K_1 = 0.208218$

$K_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2})$

$K_2 = (0.2) f(0.2 + \frac{0.2}{2}, 0.2027 + \frac{0.208}{2})$

$= (0.2) f(0.3, 0.3068)$

Solved by

$K_2 = (0.2)(1 + (0.3068)^2)$

$= 0.218825$

$K_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2})$

$= (0.2) f(0.2 + \frac{0.2}{2}, 0.2027 + \frac{0.2188}{2})$

$= (0.2) f(0.3, 0.3121125)$

$= (0.2)[1 + (0.3121125)^2]$

$K_3 = 0.2196$

$K_4 = hf(x_1 + h, y_1 + K_3)$

$= (0.2) f(0.2 + 0.2, 0.2027074 + 0.2194)$

$= (0.2)[1 + (0.422190)^2]$

$K_4 = 0.235648$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[(0.208218) + 2(0.218825) + 2(0.2194) + 0.235648]$

$K = 0.220052$

$y_2 = y_1 + K$

$y_2 = y(0.4) = 0.2027074 + 0.220052$

$y_2 = 0.42276022$

Solved by
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Q8(b) Solve $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$ $h = 0.05$ Find $y(0.1)$ by Euler's method and Taylor's method and Compare it with exact solution.

Solution:- Given $f(x,y) = x^2 + y^2$

$y(0) = 1$

$x_0 = 0$, $y_0 = 1$, $h = 0.05$

Euler's method Formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

put $n = 0$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= y_0 + h(x_0^2 + y_0^2)$$

$$= 1 + (0.05)(0^2 + 1^2)$$

$$y_1 = 1.05$$

OR $y(0.05) = 1.05$

Now $y_2 = y_1 + hf(x_1, y_1)$

$$= y_1 + h(x_1^2 + y_1^2)$$

$$= (1.05) + (0.05)\left[(0.05)^2 + (1.05)^2\right]$$

$$y_2 = 1.10525$$

OR $y(0.1) = 1.10525$

Solved by
Labeeb Ahmad

Now

Given $y' = x^2 + y^2$

$x_0 = 0$, $y_0 = 1$ $h = 0.05$

y_0

$y_0'' =$

y_0''

y_0'''

y_0'

$y_0^{(iv)}$:

$y_0^{(iv)}$

By

$y_1 = y_0$

$= 1$;

$=$

$y_1 =$

th $y(0)=1$ $y_0' = x_0^2 + y_0^2 = 0^2 + 1^2 = 1$

er's

nal $y_0'' = 2x_0 + 2y_0 \cdot y_0'$

$= 2(0) + 2(1)\cdot(1) = 0 + 2$

$y_0'' = 2$

$y_0''' = 2 + 2(y_0 \cdot y_0'' + y_0' y_0')$

$+y^2$ $= 2 + 2((1)(2) + (1)(1))$

$= 2 + 2(3)$

$h = 0.05$ $y_0''' = 8$

$y_0^{(iv)} = 2\left[(y_0 \cdot y_0''' + y_0'' \cdot y_0') + (y_0' \cdot y_0'')\right]$

$= 2\left[\{(1)(8) + (2)(1)\} + 2\{(1)(2)\}\right]$

$= 2[10 + 4]$

$y_0^{(iv)} = 28$

By Taylor series

$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{(iv)} + \cdots$

$= 1 + (0.05)(1) + \frac{(0.05)^2}{2}(2) + \frac{(0.05)^3}{3!}(8)$

$5)^2]$ $+ \frac{(0.05)^4}{4!}(28) + \cdots$

$= 1 + 0.05 + 0.0025 + 0.0001666$

ad $+ 0.0000029$

$y_1 = 1.052673$

25

Solved by
Labeeb Ahmad

Given

$y_1' = x_1^2 + y_1^2$

$= (0.05)^2 + (1.052673)^2$

$= 1.110622$

$y_1'' = 2x_1 + 2y_1 \cdot y_1'$

$= 2(0.05) + 2(1.052673)(1.110622)$

$= 2.438244$

$y_1''' = 2 + 2(y_1 y_1'' + (y_1')^2)$

$= 2 + 2[(1.052673)(2.438244) + (1.110622)^2]$

$= 2 + 2[2.56667 + 1.2334]$

$y_1''' = 9.60036$

$y_1^{(iv)} = 2[(y_1 \cdot y_1''' + y_1'' \cdot y_1') + (2y_1' \cdot y_1'')]$

$= 2[\{(1.052673)(9.60036) + (2.438244)(1.110622)\} + \{2(1.110622)(2.438244)\}]$

$= 2[12.81400977 + 5.415934]$

$= 36.459889$

BY Taylor's Series

Solved by Labeeb Ahmad

$y_2 = y_1 + hy_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \frac{h^4}{4!} \cdot y_1^{(iv)} +$

$= (1.052673) + (0.05)(1.110622) + \frac{(0.05)^2}{2!}$

$12.438244) + \frac{(0.05)^3}{} (9.60036) + \frac{(0.05)^4}{} (36.45$

$y_2 =$

$y_2 =$

$y_2 = (1.052627) + (0.055531) + (0.0030478$

$+ 0.0002000 + 0.0000949$

$y_2 = 1.111195295$

Solved by
Labeeb Ahmad

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(1.110622)]

8244)

438244)^3]

y)

nad

$+ y_i^{(m)} +$

$\Delta f)^2$

$(.05)^4 (36.45)$

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